CBSE Class 12th Mathematics Question Paper 2013 Set-3

Simplifying Test Prep

Code No. 65/1/3

Series: SKS/1

Roll No. | 9 | 3 | 3 | 4 | 0 | 8 | 0 |

Candidates must write the code on the title page of the answer-book.

- Please check that this question paper contains 8 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.
- Please write down the Serial Number of the question before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS

Time Allowed: 3 hours **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three Sections A, Band C. Section A comprises of 10 questions of one mark each, Section Q comprises of 12 questions of four marks each and Section C comprises of 07 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

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SECTION – A

Question numbers 1 to 10 carry 1 mark each.

1. Find the Cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

2. Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

3. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of x + y + z.

4. Write the degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + x^3 = 0$.

5. The amount of pollution content added in air in a city due to x-diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

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6. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

7. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

8. Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

9. Write the value of tan $\left(2\tan^{-1}\frac{1}{5}\right)$.

10. Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

SECTION- B

Question number 11 to 22 carry 4 marks each.

11. A speaks truth in 75% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? Do you think that statement of B is true?

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12. Find the coordinates of the point, where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ intersects the plane x - y + z - 5 = 0. Also find the angle between the line and the plane.

OR

Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

13. Using vectors, find the area of the triangle ABC with vertices A(1, 2, 3), B(2, -1, 4) and C(4,5,-1).

14. Evaluate: $\int_2^5 [\,|x-2| + |x-3| + |x-5|\,]\, dx.$

15. Evaluate: $\int \frac{2x^2+1}{x^2(x^2+4)}\, dx$

16. Evaluate: $\int \frac{\sin(x-a)}{\sin(x+a)}\, dx$

OR

Evaluate: $\int \frac{5x-2}{1+2x+3x^2}\, dx$

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17. Show that the function f(x) = |x- 3|, x $\in$ $\mathbb{R}$, is continuous but not differentiable at x = 3.

OR

If x = a sin t and y = a (cos t + log tan½), find $\frac{d^2y}{dx^2}$.

18. If y = log $\left[x+\sqrt{x^2+a^2}\right]$, show that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

19. Differentiate the following function with respect to x:
$(\log x)^x + x^{\log x}$

20. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$$

21. Find the value of the following :

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0 \text{ and } xy<1.$$

OR

Prove that: $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

22. Show that the function f in A = $\mathbb{R}-\left\{\frac{2}{3}\right\}$ defined as f(x) = $\frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.

SECTION – C

Question numbers 23 to 29 carry 6 marks each.

23. Assume that the chances of a patient having a heart attack are 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose anyone of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the' patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

24. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs.10,500 and Rs.9,000 respectively. To control weeds. a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects

drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment?

25. Find the coordinates of the point where the line through (3, - 4, - 5) and (2, - 3, 1) crosses the plane, passing through the points (2, 2, 1), (3, 0, 1) and (4, -1, 0).

26. Show that the differential equation $(x\, e^{x/y} + y)\, dx = xdy$ is homogeneous. Find the particular solution of this differential equation, given that x =1 when y = 1.

27. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.
OR
Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

28. Show that the height of the cylinder of maximum volume, that can be inscribed in a

29. Sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

30. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. 6,000. Three times the award money for Hard work added to that given for honesty amounts to Rs. 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.