CONTENTS

UNITS, DIMENSIONS AND MEASUREMENTS

(i) SI Units:

(a) Time-second(s);
(b) Length-metre (m);
(c) Mass-kilogram (kg);
(d) Amount of substance–mole (mol); (e) Temperature-Kelvin (K);
(f) Electric Current – ampere (A);
(g) Luminous Intensity – Candela (Cd)

(ii) Uses of dimensional analysis

(a) To check the accuracy of a given relation
(b) To derive a relative between different physical quantities
(c) To convert a physical quantity from one system to another system

$$n_1 u_2 = n_2 u_2 \quad \text{or} \quad n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \text{x} \left[\frac{L_1}{L_2}\right]^b \text{x} \left[\frac{T_1}{T_2}\right]^c$$

(iii) Mean or average value: $\overline{X} = \dfrac{X_1 - X_2 + ... + X_N}{N}$

(iv) Absolute error in each measurement: $|\Delta X_i| = |\overline{X} - X_i|$

(v) Mean absolute error: $\Delta X_m = \dfrac{\Sigma |\Delta X_i|}{N}$

(vi) Fractional error = $\dfrac{\Delta X}{X}$

(vii) Percentage error = $\dfrac{\Delta X}{X} \text{x} 100$

(viii) Combination of error: If $f = \dfrac{X^a Y^b}{Z^c}$, then maximum fractional error in f is:

$$\frac{\Delta f}{f} = |a| \frac{\Delta X}{X} + |b| \frac{\Delta Y}{Y} + |c| \frac{\Delta Z}{Z}$$

MOTION IN ONE DIMENSION & NEWTON'S LAWS OF MOTION

(i) **Displacement**: | displacement | ≤ distance covered

(ii) **Average speed**: $\bar{v} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}}$

(a) If $s_1 = s_2 = d$, then $\bar{v} = \frac{2 v_1 v_2}{v_1 + v_2}$ = Harmonic mean

(b) If $t_1 = t_2$, then $\bar{v} = \frac{v_1 + v_2}{2}$ =arithmetic mean

(iii) **Average velocity**: (a) $\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$; (b) $|\vec{v}_{av}| \le \bar{v}$

(iv) **Instantaneous velocity**: $\vec{v} = \frac{d\vec{r}}{dt}$ and $|\vec{v}|$ = v = instantaneous speed

(v) **Average acceleration**: $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

(vi) **Instantaneous acceleration**: $\vec{a} = d\vec{v}/dt$

In one – dimension, a = (dv/dt) = $\left(\frac{dv}{dx}\right)v$

(vii) **Equations of motion in one dimension**:

(a) v = u + at;

(b) $x = ut + \frac{1}{2}at^2$;

(c) $v_2 \; u_2 + 2ax$;

(d) $x = vt - \frac{1}{2}at^2$;

(e) $x = \left(\frac{v+u}{2}\right)t$;

(f) $s = x - x_0 = ut + \frac{1}{2}at^2$;

(g) $v^2 = u^2 + 2a(x - x_0)$

(viii) **Distance travelled in nth second**: $d_n = u + \frac{a}{2}(2n-1)$

(ix) **Motion of a ball**: (a) when thrown up: $h = (u^2/2g)$ and $t = (u/g)$
(b) when dropped: $v = \sqrt{(2gh)}$ and $t = \sqrt{(2h/g)}$

(x) **Resultant force**: $F = \sqrt{(F_1^2 + F_2^2 + 2F_1F_2 \cos\theta)}$

(xi) **Condition for equilibrium**: (a) $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$; (b) $F_1 + F_2 \ge F_3 \ge |F_1 - F_2|$

(xii) **Lami's Theorem**: $\frac{P}{\sin(\pi-\alpha)} = \frac{Q}{\sin(\pi-\beta)} = \frac{R}{\sin(\pi-\gamma)}$

(xiii) **Newton's second law**: $\vec{F} = m\vec{a}$; $\vec{F} = \left(d\vec{p}/dt\right)$

(xiv) **Impulse**: $\Delta\vec{p} = \vec{F}\Delta t$ and $p_2 - p_1 = \int_1^2 F dt$

(xv) **Newton's third law**:

(a) $\vec{F}_{12} = -\vec{F}_{12}$

(b) Contact force: $F_{12} = \frac{m}{M+m}F = F_{21}$

(c) Acceleration: a = $\frac{F}{M+m}$

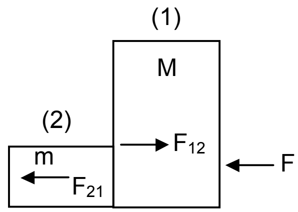

Fig. 1

(xvi) **Inertial mass**: $m_I = F/a$

(xvii) **Gravitational mass**: $m_G = \frac{F}{g} = \frac{FR^2}{GM}; m_I = m_G$

(xviii) **Non inertial frame**: If $\vec{a}_0$ be the acceleration of frame, then pseudo force $\vec{F} = -m\vec{a}_0$

Example: Centrifugal force = $\frac{mv^2}{r} = m\omega^2 r$

(xix) **Lift problems**: Apparent weight = $M(g \pm a_0)$
(+ sign is used when lift is moving up while – sign when lift is moving down)

(xx) **Pulley Problems**:

(a) For figure (2):

Tension in the string, T = $\frac{m_1 m_2}{m_1 + m_2}g$

Acceleration of the system, a = $\frac{m_2}{m_1 + m_2}g$

The force on the pulley, F = $\frac{\sqrt{2}m_1 m_2}{m_1 + m_2}g$

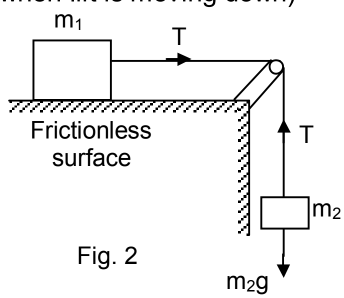

Fig. 2

(b) For figure (3):

Tension in the string, $T = \frac{2m_1 m_2}{m_1 + m_2}g$

Acceleration of the system, $a = \frac{m_2 - m_1}{m_2 + m_1}g$

The force on the pulley, $F = \frac{4m_1 m_2}{m_1 + m_2}g$

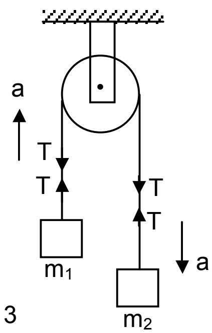

Fig. 3

VECTORS

(i) **Vector addition**: $\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$ and $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

(ii) **Unit vector**: $\hat{A} = (\vec{A}/A)$

(iii) **Magnitude**: $A = \sqrt{(A_x^2 + A_y^2 + A_z^2)}$

(iv) **Direction cosines**: $\cos\alpha = (A_x/A)$, $\cos\beta = (A_y/A)$, $\cos\gamma = (A_z/A)$

(v) **Projection**:

(a) Component of $\vec{A}$ along $\vec{B} = \vec{A}.\hat{B}$

(b) Component of $\vec{B}$ along $\vec{A} = \hat{A}.\vec{B}$

(c) If $\vec{A} = A_x\hat{i} + A_y\hat{j}$, then its angle with the x–axis is $\theta = \tan^{-1}(A_y/A_x)$

(vi) **Dot product**:

(a) $\vec{A}.\vec{B} = AB\cos\theta$, (b) $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

(vii) **Cross product**:

(a) $\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$;

(b) $\vec{A} \times \vec{A} = 0$;

(c) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

(viii) **Examples**:

(a) $W = \vec{F}.\vec{r}$; (b) $P = \vec{F}.\vec{v}$; (c) $\phi_E = \vec{E}.\vec{A}$; (d) $\phi_B = \vec{B}.\vec{A}$;

(e) $\vec{v} = \vec{w} \times \vec{r}$; (f) $\vec{\tau} = \vec{\tau} \times \vec{F}$; (g) $\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$

(ix) **Area of a parallelogram**: Area = $|\vec{A} \times \vec{B}|$

(x) **Area of a triangle**: Area = $\frac{1}{2}|\vec{A} \times \vec{B}|$

(xi) **Gradient operator**: $\overline{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

(xii) **Volume of a parallelopiped**: $V = \vec{A}.\left(\vec{B} \times \vec{C}\right)$

CIRCULAR MOTION, RELATIVE MOTION & PROJECTILE MOTION

(i) **Uniform Circular Motion**:
- (a) $v = \omega r$;
- (b) $a = (v^2/r) = \omega^2 r$;
- (c) $F = (mv^2/r)$;
- (d) $\vec{r} \cdot \vec{v} = 0$;
- (e) $\vec{v} \cdot \vec{a} = 0$

(ii) **Cyclist taking a turn**: $\tan\theta = (v^2/rg)$

(iii) **Car taking a turn on level road**: $v = \sqrt{(\mu_s rg)}$

(iv) **Banking of Roads**: $\tan\theta = v^2/rg$

(v) **Air plane taking a turn**: $\tan\theta = v^2/r\,g$

(vi) **Overloaded truck**:
- (a) $R_{inner\ wheel} < R_{outer\ wheel}$
- (b) maximum safe velocity on turn, $v = \sqrt{(gdr/2h)}$

(vii) **Non–uniform Circular Motion**:
- (a) Centripetal acceleration $a_r = (v^2/r)$;
- (b) Tangential acceleration $a_t = (dv/dt)$;
- (c) Resultant acceleration $a = \sqrt{(a_r^2 + a_t^2)}$

(viii) **Motion in a vertical Circle**:
- (a) For lowest point A and highest point B, $T_A - T_B = 6\,mg$; $v^2_A = v^2_B + 4g\ell$; $v_A \geq \sqrt{(5g\ell)}$; and $v_B \geq \sqrt{(g\ell)}$
- (b) Condition for Oscillation: $v_A < \sqrt{(2g\ell)}$
- (c) Condition for leaving Circular path: $\sqrt{(2g\ell)} < v_A < \sqrt{(5g\ell)}$

(ix) **Relative velocity**: $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

(x) **Condition for Collision of ships**: $(\vec{r}_A - \vec{v}_B)\,x\,(\vec{v}_A - \vec{v}_B) = 0$

(xi) **Crossing a River**:
- (a) Beat Keeps its direction perpendicular to water current
- (1) $v_R = \sqrt{((v_w^2 + v_b^2)}$; (2) $\theta = \tan^{-1}(v_w / v_b)$;
- (3) $t = (x/v_b)$ (it is minimum) (4) Drift on opposite bank = $(v_w/v_b)x$
- (b) Boat to reach directly opposite to starting point:

  (1) $\sin\theta = (v_w/v_b)$; (2) $v_{resultant} = v_b \cos\theta$; (3) $t = \dfrac{x}{v_b \cos\theta}$

(xii) **Projectile thrown from the ground**:
- (a) equation of trajectory: $y = x\tan\theta - \dfrac{g x^2}{2u^2 \cos^2\theta}$
- (b) time of flight: $T = \dfrac{2\,u \sin\theta}{g}$
- (c) Horizontal range, $R = (u^2 \sin 2\theta/g)$
- (d) Maximum height attained, $H = (u^2 \sin^2\theta/2g)$

(e) Range is maximum when $\theta = 45^0$
(f) Ranges are same for projection angles θ and $(90^0 - \theta)$
(g) Velocity at the top most point is $= u \cos\theta$
(h) $\tan\theta = gT^2/2R$
(i) $(H/T^2) = (g/8)$

(xiii) **Projectile thrown from a height h in horizontal direction**:
(a) $T = \sqrt{(2h/g)}$;
(b) $R = v\sqrt{(2h/g)}$;
(c) $y = h - (gx^2/2u^2)$

(d) Magnitude of velocity at the ground $= \sqrt{(u^2 + 2gh)}$
(e) Angle at which projectiles strikes the ground, $\theta = \tan^{-1}\sqrt{\frac{2gh}{u}}$

(xiv) **Projectile on an inclined plane**:
(a) Time of flight, $T = \frac{2u \sin(\theta - \theta_0)}{g \cos\theta_0}$

(b) Horizontal range, $R = \frac{2u^2 \sin(\theta - \theta_0)\cos\theta}{g \cos^2\theta_0}$

FRICTION

(i) **Force of friction**:
  (a) $f_s \le \mu_s N$ (self adjusting); $(f_s)_{max} = \mu_s N$
  (b) $\mu_k = \mu_k N$ (μ_k = coefficient of kinetic friction)
  (c) $\mu_k < \mu_s$

(ii) **Acceleration on a horizontal plane**: $a = (F - \mu_k N)/M$

(iii) **Acceleration of a body sliding on an inclined plane**: $a = g \sin\theta\,(1 - \mu_k \cot t^2)$

(iv) **Force required to balance an object against wall**: $F = (Mg/\mu_s)$

(v) **Angle of friction**: $\tan\theta = \mu_s$ (μ_s = coefficient of static friction)

DYNAMICS OF RIGID BODIES

(i) **Average angular velocity**: $\overline{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

(ii) **Instantaneous angular velocity**: $\omega = (d\theta/dt)$

(iii) **Relation between v, ω and r** : $v = \omega r$; In vector form $\vec{v} = \vec{\omega} \times \vec{r}$; In general form, $v = \omega r \sin\theta$

(iv) **Average angular acceleration**: $\overline{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$

(v) **Instantaneous angular acceleration**: $\alpha = (d\omega/dt) = (d^2\theta/dt^2)$

(vi) **Relation between linear and angular acceleration**:
  (a) $a_T = \alpha r$ and $a_R = (v^2/r) = \omega^2 R$
  (b) Resultant acceleration, $a = \sqrt{(a_T^2 + a_R^2)}$
  (c) In vector form,
  $\vec{a} = \vec{a}_T + \vec{a}_R$, where $\vec{a}_T = \vec{\alpha} \times \vec{r}$ and $\vec{a}_R = \vec{\omega} \times \vec{u} = \vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$

(vii) **Equations for rotational motion**:
  (a) $\omega = \omega_0 + \alpha t$;
  (b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$;
  (c) $\omega^2 - \omega_0^2 = 2\alpha\theta$

(viii) **Centre of mass**: For two particle system:

  (a) $x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$;
  (b) $v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$
  (c) $a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

  Also $v_{CM} = \frac{dx_{CM}}{dt}$ and $a_{CM} = \frac{dv_{CM}}{dt} = \frac{d^2 x_{CM}}{dt^2}$

(ix) **Centre of mass**: For many particle system:

(a) $X_{CM} = \frac{\Sigma m_i x_i}{M}$;

(b) $\vec{r}_{CM} = \frac{\Sigma m_i \vec{r}_i}{M}$;

(c) $\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt}$;

(d) $\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt}$;

(e) $\vec{P}_{CM} = M\vec{v}_{CM} = \Sigma m_i \vec{v}_i$;

(f) $\vec{F}_{ext} = M\vec{a}_{CM} = \Sigma m_i \vec{a}_i = \Sigma \vec{F}_i$. If $\vec{F}_{ext} = 0, \vec{a}_{CM} = 0, \vec{V}_{CM} = \text{constant}$;

(g) Also, moment of masses about CM is zero, i.e., $\Sigma m_i \vec{r}_i = 0$ or $m_1 r_1 = m_2 r_2$

(x) **Moment of Inertia**: (a) $I = \Sigma m_i r_i^2$
(b) $I = \mu r^2$, where $\mu = m_1 m_2/(m_1 + m_2)$

(xi) **Radius of gyration**: (a) $K = \sqrt{(I/M)}$; (b) $K = \sqrt{[(r_1^2 + r_2^2 + \ldots + r_n^2)/n]}$ = root mean square distance.

(xii) **Kinetic energy of rotation**: $K = \frac{1}{2} I\omega^2$ or $I = (2K/\omega^2)$

(xiii) **Angular momentum**: (a) $\vec{L} = \vec{r} \times \vec{p}$; (b) $L = rp\sin\theta$; (c) mvd

(xiv) **Torque**: (a) $\vec{\tau} = \vec{r} \times \vec{F}$; (b) $\tau = rF\sin\theta$

(xv) **Relation between τ and L**: $\vec{\tau} = \left(d\vec{L}/dt\right)$;

(xvi) **Relation between L and I**: (a) $L = I\omega$; (b) $K = \frac{1}{2} I\omega^2 = L^2/2I$

(xvii) **Relation between τ and α**:
(a) $\tau = I\alpha$,
(b) If $\tau = 0$, then $(dL/dt)=0$ or L=constant or, $I\omega$=constant i.e., $I_1\omega_1 = I_2\omega_2$
(Laws of conservation of angular momentum)

(xviii) **Angular impulse**: $\Delta\vec{L} = \vec{\tau}\Delta t$

(xix) **Rotational work done**: $W = \int \tau\, d\theta = \tau_{av}\theta$

(xx) **Rotational Power**: $P = \vec{\tau} \cdot \vec{\omega}$

(xxi) **(a) Perpendicular axes theorem**: $I_z = I_x + I_y$
(b) Parallel axes theorem: $I = I_c + Md^2$

(xxii) Moment of Inertia of some objects

(a) Ring: $I = MR^2$ (axis); $I = \frac{1}{2} MR^2$ (Diameter);
$I = 2\,MR^2$ (tangential to rim, perpendicular to plane);
$I = (3/2)\,MR^2$ (tangential to rim and parallel to diameter)

(b) Disc: $I = \frac{1}{2}MR^2$ (axis); $I = \frac{1}{4}MR^2$ (diameter)

(c) Cylinder: $I = \frac{1}{2}MR^2$ (axis)

(d) Thin rod: $I = (ML^2/12)$ (about centre); $I = (ML^2/3)$ (about one end)

(e) Hollow sphere : $I_{dia} = (2/3)\, MR^2$; $I_{tangential} = (5/3)\, MR^2$

(f) Solid sphere: $I_{dia} = (2/5)\, MR^2$; $I_{tangential} = (7/5)\, MR^2$

(g) Rectangular: $I_C = \frac{M(\ell^2 + b^2)}{12}$ (centre)

(h) Cube: $I = (1/6)\, Ma^2$

(i) Annular disc: $I = (1/2)\, M\, (R_1^2 + R_2^2)$

(j) Right circular cone: $I = (3/10)\, MR^2$

(k) Triangular lamina: $I = (1/6)\, Mh^2$ (about base axis)

(l) Elliptical lamina: $I = (1/4)\, Ma^2$ (about minor axis) and $I = (1/4)\, Mb^2$ (about major axis)

(xxiii) **Rolling without slipping on a horizontal surface**:

$$K = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MV^2\left[1 + \frac{K^2}{R^2}\right] \qquad (\because V = R\omega \text{ and } I = MK^2)$$

For inclined plane

(a) Velocity at the bottom, $v = \sqrt{2gh \Big/ \left(1 + \frac{K^2}{R^2}\right)}$

(b) Acceleration, $a = g\sin\theta \Big/ \left(1 + \frac{K^2}{r^2}\right)$

(c) Time taken to reach the bottom, $t = \sqrt{2s\left(1 + \frac{K^2}{R^2}\right) \Big/ g\sin\theta}$

(xxiv) **Simple pendulum**: = $T = 2\pi\sqrt{(L/g)}$

(xxv) **Compound Pendulum**: $T = 2\pi\sqrt{(I/Mg\,\ell)}$, where $\ell = M(K^2 + \ell^2)$
Minimum time period, $T_0 = 2\pi\sqrt{(2K/g)}$

(xxvi) **Time period for disc**: $T = 2\pi\sqrt{(3R/2g)}$
Minimum time period for disc, $T = 2\pi\sqrt{(1.414R/g)}$

(xxvii) **Time period for a rod of length L pivoted at one end**: $T = 2\pi\sqrt{(2L/3g}$

The heights by great men reached and kept…
…were not attained by sudden flight,

but they, while their companions slept…
…were toiling upwards in the night.

CONSERVATION LAWS AND COLLISIONS

(i) **Work done**: (a) $W = \vec{F}.\vec{d}$; (b) $W = Fd\cos\theta$; (c) $W = \int_{x_1}^{x_2}$

(ii) **Conservation forces**: (a) $\int_a^b \vec{F}.d\vec{r}$ (Path 1) (b) $\int_a^b \vec{F}.d\vec{r}$ (Path 2); (c) $\oint \vec{F}.d\vec{r} = 0$ (closed path)

For conservative forces, one must have: $\overline{\nabla} \times \vec{F} = 0$

(iii) **Potential energy**: (a) $\overline{\nabla}U = -W$; (b) $F = -(dU/dX)$; (c) $\vec{F} = -\overline{\nabla}U$

(iv) **Gravitational potential energy**: (a) U = mgh ; (b) $U = -\frac{GMm}{(R+h)}$

(v) **Spring potential energy**: (a) $U = \frac{1}{2}Kx^2$; (b) $\Delta U = \frac{1}{2}K\left(x_2^2 - x_1^2\right)$

(vi) **Kinetic energy**: (a) $\Delta K = W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$; (b) $K = \frac{1}{2}mv^2$

(vii) **Total mechanical energy**: = E = K + U

(viii) **Conservation of energy**: $\Delta K = -\Delta U$ or, $K_f + U_f = K_i + U_i$

In an isolated system, E_{total} = constant

(ix) **Power**: (a) P = (dw/dt) ; (b) P = (dw/dt) ; (c) $P = \vec{F}.\vec{v}$

(x) **Tractive force**: F = (P/v)

(xi) **Equilibrium Conditions**:
- (a) For equilibrium, (dU/dx) = 0
- (b) For stable equilibrium: U(x) = minimum, (dU/dx) = 0 and (d^2U/dx^2) is positive
- (c) For unstable equilibrium: U(x) = maximum, (dU/dx) = 0 and (d^2U/dx^2) is negative
- (d) For neutral equilibrium: U(x) = constant, (dU/dx) = 0 and (d^2U/dx^2) is zero

(xii) **Velocity of a particle in terms of U(x)**: $v = \pm\sqrt{\frac{2}{m}[E - U(x)]}$

(xiii) **Momentum**:

(a) $\vec{p} = m\vec{v}$; (b) $\vec{F} = \left(d\vec{p}/dt\right)$,

(b) Conservation of momentum: If $\vec{F}_{net} = 0$, then $\vec{p}_f = \vec{p}_i$,

(c) Recoil speed of gun, $v_G = \frac{m_B}{m_G} \times v_B$

(xiv) **Impulse**: $\Delta\vec{p} = \vec{F}_{av}\Delta t$

(xv) **Collision in one dimension**:
- (a) Momentum conservation : $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- (b) For elastic collision, e = 1 = coefficient of restitution
- (c) Energy conservation: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

(d) Velocities of 1st and 2nd body after collision are:

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_2 + m_1}\right)u_2; \quad v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_2 + m_1}\right)u_2$$

(e) If $m_1 = m_2 = m$, then $v_1 = u_2$ and $v_2 = u_1$

(f) Coefficient of restitution, $e = (v_2 – v_1/u_1 = u_2)$

(g) e = 1 for perfectly elastic collision and e=0 for perfectly inelastic collision. For inelastic collision 0 < e < 1

(xvi) Inelastic collision of a ball dropped from height h_0

(a) Height attained after nth impact, $h_n = e^{2n}h_0$

(b) Total distance traveled when the ball finally comes to rest, $s = h_0\,(1+e^2)/(1–e^2)$

(c) Total time taken, $t = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$

(xvii) **Loss of KE in elastic collision**: For the first incident particle

$$\frac{K_f}{K_i} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \text{ and } \frac{\Delta K_{lost}}{K_i} = \frac{4m_1m_2}{(m_1 + m_2)^2}; \text{ If } m_1 = m_2, \frac{\Delta K_{lost}}{K_i} = 100\%$$

(xviii) **Loss of KE in inelastic collision**: $\Delta K_{lost} = K_i – K_f = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 – u_2)^2\,(1–e^2)$

Velocity after inelastic collision (with target at rest)

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 \text{ and } v_2 = \frac{m_1(1+e)}{m_1 + m_2}u_1$$

(xix) **Oblique Collision (target at rest)**:

$m_1u_1 = m_1v_1 \cos\theta_1 + m_2v_2 \cos\theta_2$ and $m_1v_1 \sin\theta_1 = m_2v_2 \sin\theta_2$

Solving, we get: $m_1u_1^2 = m_1v_1^2 + m_2v_2^2$

(xx) **Rocket equation**: (a) $M\frac{dV}{dt} = -v_{el}\frac{dM}{dt}$

(b) $V = – v_{rel} \log_e \left(\frac{M_0 - m_b}{M_0}\right)$ [M_0 = original mass of rocket plus fuel and m_b = mass of fuel burnt]

(c) If we write $M = M_0 – m_b$ = mass of the rocket and full at any time, than velocity of rocks at that time is:
$V = v_{rel} \log_e (M_0/M)$

(xxi) **Conservation of angular momentum**:

(a) If $\tau_{ext} = 0$, then $L_f = L_i$

(b) For planets, $\frac{v_{max}}{v_{min}} = \frac{r_{max}}{r_{min}}$

(c) Spinning skater, $I_1\omega_1 = I_2W_2$ or $\omega_f = \omega_i\left(\frac{I_i}{I_f}\right)$

SIMPLE HARMONIC MOTION AND LISSAJOUS FIGURES

(i) Simple Harmonic Motion:

(a) $F = -Kx$;

(b) $a = -\frac{K}{m}x$ or $a = -\omega^2 x$, where $\omega = \sqrt{(K/m)}$;

(c) $F_{max} = \pm KA$ and $a_{max} = \pm\omega^2 A$

(ii) **Equation of motion**: $\frac{d^2x}{dt^2} + \omega^2 x = 0$

(iii) **Displacement**: $x = A \sin(\omega t + \phi)$

(a) If $\phi = 0$, $x = A \sin \omega t$;

(b) If $\phi = \pi/2$, $x = A \cos \omega t$

(c) If $x = C \sin \omega t + D \cos \omega t$, then $x = A \sin(\omega t + \phi)$ with $A = \sqrt{(C^2+D^2)}$ and $\phi = \tan^{-1}(D/C)$

(iv) **Velocity**:

(a) $v = A\,\omega \cos(\omega + \phi)$;

(b) If $\phi=0$, $v = A\,\omega \cos \omega t$;

(c) $v_{max} = \pm\omega A$

(d) $v = \pm\,\omega\sqrt{(A^2 - x^2)}$;

(e) $\frac{x^2}{A^2} + \frac{v^2}{\omega^2 A^2} = 1$

(v) **Acceleration**:

(a) $a = -\omega^2 x = -\omega^2 A \sin(\omega t+\phi)$;

(b) If $\phi=0$, $a = -\omega^2 A \sin \omega t$

(c) $|a_{max}| = \omega^2 A$;

(d) $F_{max} = \pm m\,\omega^2 A$

(vi) **Frequency and Time period**:

(a) $\omega = \sqrt{(K/m)}$;

(b) $f = \frac{1}{2\pi}\sqrt{(K/m)}$;

(c) $T = 2\pi\sqrt{\frac{m}{K}}$

(vii) **Energy in SHM: Potential Energy**:

(a) $U = \frac{1}{2}Kx^2$;

(b) $F = -\frac{dU}{dx}$;

(c) $U_{max} = \frac{1}{2}m\omega^2 A^2$;

(d) $U = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$

(viii) **Energy in SHM: Kinetic energy**:

(a) $K = \frac{1}{2}mv^2$;

(b) $K = \frac{1}{2}m\omega^2 (A^2 - x^2)$;

(c) $K = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$;

(d) $K_{max} = \frac{1}{2}m\omega^2 A^2$

(ix) **Total energy**:

(a) $E = K + U$ = conserved;

(b) $E = (1/2)\, m\omega^2 A^2$;

(c) $E = K_{max} = U_{max}$

(x) **Average PE and KE**:

(a) $\langle U \rangle = (1/4)\, m\omega^2 A^2$;

(b) $\langle K \rangle = (1/4)\, m\omega^2 A^2$;

(c) $(E/2) = \langle U \rangle = \langle K \rangle$

(xi) Some relations:

(a) $\omega = \sqrt{\dfrac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$; (b) $T = 2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$; (c) $A = \sqrt{\dfrac{(v_1 x_2)^2 - (v_2 x_1)^2}{v_1^2 - v_2^2}}$

(xii) Spring– mass system:

(a) $mg = Kx_0$;

(b) $T = 2\pi\sqrt{\dfrac{m}{K}} = 2\pi\sqrt{\dfrac{x_0}{g}}$

(xiii) **Massive spring**: $T = 2\pi\sqrt{\dfrac{m + (m_s/3)}{K}}$

(xiv) **Cutting a spring**:

(a) $K' = nK$;

(b) $T' = T_0/\sqrt{(n)}$;

(c) $f' = \sqrt{(n)}\, f_0$

(d) If spring is cut into two pieces of lengths ℓ_1 and ℓ_2 such that $\ell_1 = n\ell_2$, then $K_1 = \left(\dfrac{n+1}{n}\right)K$, $K_2 = (n+1)\,K$ and $K_1\ell_1 = K_2\ell_2$

(xv) **Springs in parallel**:

(a) $K = K_1 + K_2$;

(b) $T = 2\pi\sqrt{[m/(K_1 + K_2)]}$

(c) If $T_1 = 2\pi\sqrt{(m/K_1)}$ and $T_2 = 2\pi\sqrt{(m/K_2)}$, then for the parallel combination:

$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} \quad \text{or} \quad T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}} \quad \text{and} \quad \omega^2 = \omega_1^2 + \omega_2^2$$

(xvi) **Springs in series**:

(a) $K_1 x_1 = K_2 x_2 = Kx = F$ applied

(b) $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$ or $K = \dfrac{K_1 K_2}{K_1 + K_2}$

(c) $\dfrac{1}{\omega^2} = \dfrac{1}{\omega_1^2} + \dfrac{1}{\omega_2^2}$ or $T^2 - T_1{}^2 + T_2^2$

(d) $T = 2\pi\sqrt{\dfrac{m(K_1 + K_2)}{K_1 K_2}}$ or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{K_1 K_2}{m(K_1 + K_2)}}$

(xvii) **Torsional pendulum**:

(a) $I\alpha = \tau - C\theta$ or $\dfrac{d^2\theta}{dt^2} + \dfrac{C}{I}\theta = 0$;

(b) $\theta = \theta_0 \sin(\omega t + \phi)$;

(c) $\omega = \sqrt{(C/I)}$;

(d) $f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$;

(e) $T = 2\pi\sqrt{(I/C)}$, where $C = \pi\eta r^4/2\ell$

(xviii) **Simple pendulum**:

(a) $I\alpha = \tau = -mg\ell \sin\theta$ or $\frac{d^2\theta}{dt^2} + \left(\frac{g}{\ell}\right)\sin\theta = 0$ or $\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\theta = 0$;

(b) $\omega = \sqrt{(g/\ell)}$;

(c) $f = \frac{1}{2\pi}\sqrt{(g/\ell)}$;

(d) $T = 2\pi\sqrt{(\ell/g)}$

(xix) **Second pendulum**:

(a) T = 2 sec ;

(b) $\ell = 99.3$ cm

(xx) **Infinite length pendulum**:

(a) $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{\ell} + \frac{1}{R_e}\right)}}$;

(b) $T = 2\pi\sqrt{\frac{R_e}{g}}$ (when $\ell \to \infty$)

(xxi) **Anharmonic pendulum**: $T \cong T_0\left(1 + \frac{\theta_0^2}{16}\right) \cong T_0\left(1 + \frac{A^2}{16\ell^2}\right)$

(xxii) **Tension in string of a simple pendulum**: $T = (3\,mg\cos\theta - 2\,mg\cos\theta_0)$

(xxiii) **Conical Pendulum**:

(a) $v = \sqrt{(gR\tan\theta)}$;

(b) $T = 2\pi\sqrt{(L\cos\theta/g)}$

(xxiv) **Compound pendulum**: $T = 2\pi\sqrt{\frac{(\ell + K^2/\ell)}{2}}$

(a) For a bar: $T = 2\pi\sqrt{(2L/3g)}$;

(b) For a disc : $T = 2\pi\sqrt{(3R/2g)}$

(xxv) **Floating cylinder**:

(a) $K = A\rho g$;

(b) $T = 2\pi\sqrt{(m/A\rho g)} = 2\pi\sqrt{(Ld/\rho g)}$

(xxvi) **Liquid in U–tube**:

(a) $K = 2A\rho g$ and $m = AL\rho$;

(b) $T = 2\pi\sqrt{(L/2g)} = 2\pi\sqrt{(h/g)}$

(xxvii) **Ball in bowl**: $T = 2\pi\sqrt{[(R - r)/g]}$

(xxviii) **Piston in a gas cylinder**:

(a) $K = \frac{A^2E}{V}$;

(b) $T = 2\pi\sqrt{\frac{mV}{A^2E}}$;

(c) $T = 2\pi\sqrt{\frac{V_m}{A^2 P}}$ (E–P for Isothermal process);

(d) $T = 2\pi\sqrt{\frac{V_m}{A^2 \gamma P}}$ (E = γ P for adiabatic process)

(xxix) **Elastic wire**:

(a) $K = \frac{AY}{\ell}$;

(b) $T = 2\pi\sqrt{\frac{\ell m}{AY}}$

(xxx) **Tunnel across earth**: $T = 2\pi\sqrt{(R_e/g)}$

(xxxi) **Magnetic dipole in magnetic field**: $T = 2\pi\sqrt{(I/MB)}$

(xxxii) **Electrical LC circuit**: $T = 2\pi\sqrt{LC}$ or $f = \frac{1}{2\pi\sqrt{LC}}$

(xxxiii) **Lissajous figures** –

Case (a): $\omega_1 = \omega_2 = \omega$ or $\omega_1 : \omega_2 = 1 : 1$

General equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi = \sin^2\phi$

For $\phi = 0$: $y = (b/a)\,x$; straight line with positive slope

For $\phi = \pi/4$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{\sqrt{2}xy}{ab} = \frac{1}{2}$; oblique ellipse

For $\phi = \pi/2$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; symmetrical ellipse

For $\phi = \pi$: $y = -(b/a)\,x$; straight line with negative slope.

Case (b): For $\omega_1 : \omega_2 = 2:1$ with $x = a\sin(2\omega t + \phi)$ and $y = b\sin\omega t$

For $\phi = 0, \pi$: Figure of eight

For $\phi = \frac{\pi}{4}, \frac{3\pi}{4}$: Double parabola

For $\phi = \frac{\pi}{2}, \frac{3\pi}{2}$: Single parabola

GRAVITATION

(i) **Newton's law of gravitation**:

(a) $F = G\, m_1 m_2/r^2$; (b) $a = 6.67 \times 10^{-11}$ K.$m^2/(kg)^2$; (c) $\frac{dF}{F} = -\frac{2\,dr}{r}$

(ii) **Acceleration due to gravity** (a) $g = GM/R^2$; (b) Weight $W = mg$

(iii) **Variation of g**:

(a) due to shape ; $g_{equator} < g_{pole}$

(b) due to rotation of earth: (i) $g_{pole} = GM/R^2$ (No effect)

(ii) $g_{equator} = \frac{GM}{R^2} - \omega^2 R$

(iii) $g_{equator} < g_{pole}$

(iv) $\omega^2 R = 0.034\ m/s^2$

(v) If $\omega \cong 17\ \omega_0$ or $T = (T_0/17) = (24/17)h = 1.4$ h, then object would float on equator

(c) At a height h above earth's surface $g' = g\left(1 - \frac{2h}{g}\right)$, if $h << R$

(d) At a depth of below earth's surface: $g' = g\left(1 - \frac{d}{R}\right)$

(iv) **Acceleration on moon**: $g_m = \frac{GM_m}{R_m^2} \cong \frac{1}{6} g_{earth}$

(v) **Gravitational field**: (a) $\vec{g} = -\frac{GM}{r^2}\hat{r}$ (outside); (b) $\vec{g} = -\frac{GM}{R^3} r\hat{r}$ (inside)

(vi) **Gravitational potential energy of mass m**:

(a) At a distance r : $U(r) = -GMm/r$

(b) At the surface of the earth: $U_0 = -GMm/R$

(c) At any height h above earth's surface: $U - U_0 = mgh$ (for $h << R$)

or $U = mgh$ (if origin of potential energy is shifted to the surface of earth)

(vii) **Potential energy and gravitational force**: $F = -(dU/dR)$

(viii) **Gravitational potential**: $V(r) = -GM/r$

(ix) **Gravitational potential energy of system of masses**:

(a) Two particles: $U = -Gm_1m_2/r$

(b) Three particles: $U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_2m_3}{r_{23}}$

(x) **Escape velocity**:

(a) $v_e = \sqrt{\frac{2GM}{R}}$ or $v_e = \sqrt{(2gR)} = \sqrt{(gD)}$

(b) $v_e = R\sqrt{\frac{8\pi G\rho}{3}}$

(xi) **Maximum height attained by a projectile**:

$$h = \frac{R}{(v_e/v)^2 - 1} \quad \text{or} \quad v = v_e\sqrt{\frac{h}{R+h}} \cong v_e\sqrt{\frac{h}{R}} \quad \text{(if } h << R\text{)}$$

(xii) **Orbital velocity of satellite**:

(a) $v_0 = \sqrt{\frac{GM}{r}}$; (b) $v_0 = v_e\sqrt{\frac{R}{2(R+h)}}$; (c) $v_0 \cong v_e/\sqrt{2}$ (if h<<R)

(xiii) **Time period of satellite**: (a) $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$; (b) $T = 2\pi\sqrt{\frac{R}{g}}$ (if $h << R$)

(xiv) **Energy of satellite**: (a) Kinetic energy K = $\frac{1}{2}mv_0^2 = \frac{1}{2}\frac{GMm}{r}$

(b) Potential energy U = $-\frac{GMm}{r} = -2K$;

(c) Total energy E=K + U= $-\frac{1}{2}\frac{GMm}{r}$;

(d) E = U/2 = – K ; (e) BE = –E = $\frac{1}{2}\frac{GMm}{r}$

(xv) **Geosynchronous satellite**: (a) T = 24 hours ; (b) $T^2 = \frac{4\pi^2}{GM}(R+h)^3$;

(c) $h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$; (d) $h \cong 36{,}000$ km.

(xvi) **Kepler's law**:

(a) Law of orbits: Orbits are elliptical

(b) Law of areas: Equal area is swept in equal time

(c) Law of period: $T^2 \propto r^3$; $T^2 = (4\pi^2/GM)r^3$

The most powerful weapon on earth is human soul on fire!

SURFACE TENSION

(i) (a) $T = \frac{\text{Force}}{\text{Length}} = \frac{F}{\ell}$; (b) $T = \frac{\text{Surface energy}}{\text{Surface area}} = \frac{W}{A}$

(ii) **Combination of n drops into one big drop**: (a) $R = n^{1/3}r$

(b) $E_i = n(4\pi r^2T)$, $E_f = 4\pi R^2T$, $(E_f/E_i) = n^{-1/3}$, $\frac{\Delta E}{E_i} = \left(1 - \frac{1}{n^{1/3}}\right)$

(c) $\Delta E = 4\pi R^2T\,(n^{1/3} - 1) = 4\pi R^3T\left(\frac{1}{r} - \frac{1}{R}\right)$

(iii) **Increase in temperature**: $\Delta\theta = \frac{3T}{\rho s}\left(\frac{1}{r} - \frac{1}{R}\right)$ or $\frac{3T}{\rho sJ}\left(\frac{1}{r} - \frac{1}{R}\right)$

(iv) **Shape of liquid surface**:

(a) Plane surface (as for water – silver) if $F_{adhesive} > \frac{F_{cohesive}}{\sqrt{2}}$

(b) Concave surface (as for water – glass) if $F_{adhesive} > \frac{F_{cohesive}}{\sqrt{2}}$

(c) Convex surface (as for mercury–glass) if $F_{adhesive} < \frac{F_{cohesive}}{\sqrt{2}}$

(v) **Angle of contact**:

(a) Acute: If $F_a > F_c/\sqrt{2}$;

(b) obtuse: if $F_a < F_c/\sqrt{2}$;

(c) $\theta_c = 90^0$: if $F_a = F_c\sqrt{}/2$

(d) $\cos\theta_c = \frac{T_{sa} - T_{s\ell}}{T_{\ell a}}$, (where T_{sa}, $T_{s\ell}$ and $T_{\ell a}$ represent solid-air, solid- liquid and liquid-air surface tensions respectively). Here θ_c is acute if $T_{s\ell} < T_{sa}$ while θ_c is obtuse if $T_{s\ell} > T_{sa}$

(vi) **Excess pressure**:

(a) General formula: $P_{excess} = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(b) For a liquid drop: $P_{excess} = 2T/R$

(c) For an air bubble in liquid: $P_{excess} = 2T/R$

(d) For a soap bubble: $P_{excess} = 4T/R$

(e) Pressure inside an air bubble at a depth h in a liquid: $P_{in} = P_{atm} + hdg + (2T/R)$

(vii) **Forces between two plates with thin water film separating them**:

(a) $\Delta P = T\left(\frac{1}{r} - \frac{1}{R}\right)$;

(b) $F = AT\left(\frac{1}{r} - \frac{1}{R}\right)$;

(c) If separation between plates is d, then $\Delta P = 2T/d$ and $F = 2AT/d$

(viii) **Double bubble**: Radius of Curvature of common film $R_{common} = \frac{rR}{R - r}$

(ix) **Capillary rise**:

(a) $h = \frac{2T\cos\theta}{rdg}$;

(b) $h = \frac{2T}{rdg}$ (For water $\theta = 0^0$)

(c) If weight of water in meniseus is taken into account then T = $\frac{rdg\left(h+\frac{r}{3}\right)}{2\cos\theta}$

(d) Capillary depression, $h - \frac{2T\cos(\pi-\theta)}{rdg}$

(x) **Combination of two soap bubbles**:

(a) If ΔV is the increase in volume and ΔS is the increase in surface area, then $3P_0\Delta V + 4T\Delta S = 0$ where P_0 is the atmospheric pressure

(b) If the bubbles combine in environment of zero outside pressure isothermally, then $\Delta S = 0$ or $R_3 = \sqrt{\left(R_1^2 + R_2^2\right)}$

ELASTICITY

(i) **Stress**: (a) Stress = [Deforming force/cross–sectional area];
(b) Tensile or longitudinal stress = $(F/\pi r^2)$;
(c) Tangential or shearing stress = (F/A);
(d) Hydrostatic stress = P

(ii) **Strain**: (a) Tensile or longitudinal strain = $(\Delta L/L)$;
(b) Shearing strain = ϕ;
(c) Volume strain = $(\Delta V/V)$

(iii) **Hook's law**:

(a) For stretching: Stress = Y x Strain or $Y = \frac{FL}{A(\Delta L)}$

(b) For shear: Stress = η x Strain or $\eta = F/A\phi$

(c) For volume elasticity: Stress = B x Strain or $B = -\frac{P}{(\Delta V/V)}$

(iv) **Compressibility**: K = (1/B)

(v) **Elongation of a wire due to its own weight**: $\Delta L = \frac{1}{2}\frac{MgL}{YA} = \frac{1}{2}\frac{L^2\rho g}{Y}$

(vi) **Bulk modulus of an idea gas**: $B_{isothermal} = P$ and $B_{adiabatic} = \gamma P$ (where $\gamma = C_p/C_v$)

(vii) Stress due to heating or cooling of a clamped rod
Thermal stress = $Y\alpha(\Delta t)$ and force = $YA\alpha(\Delta t)$

(viii) **Torsion of a cylinder**:

(a) $r\theta = \ell\phi$ (where θ = angle of twist and ϕ = angle of shear);
(b) restoring torque $\tau = c\theta$
(c) restoring Couple per unit twist, $c = \pi\eta r^4/2\ell$ (for solid cylinder)
and $C = \pi\eta(r_2^4 - r_1^4)/2\ell$ (for hollow cylinder)

(ix) **Work done in stretching**:

(a) W = $\frac{1}{2}$ x stress x strain x volume = $\frac{1}{2}Y$ (strain)^2 x volume = $\frac{1}{2}\frac{(\text{stress})^2}{Y}$ x volume

(b) Potential energy stored, U = W = $\frac{1}{2}$ x stress x strain x volume

(c) Potential energy stored per unit volume, u = $\frac{1}{2}$ x stress x strain

(x) **Loaded beam**:

(a) depression, $\delta = \frac{W\ell^3}{4Ybd^3}$ (rectangular)

(b) Depression, $\delta = \frac{W\ell^3}{12Y\pi r^2}$ (cylindrical)

(xi) **Position's ratio**:

(a) Lateral strain = $-\frac{\Delta D}{D} = \frac{-\Delta r}{r}$

(b) Longitudinal strain = $(\Delta L/L)$

(c) Poisson's ratio $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\Delta r/r}{\Delta L/L}$

(d) Theoretically, $-1 < \sigma < 0.5$ but experimentally $\sigma \cong 0.2 - 0.4$

(xii) **Relations between Y, η, B and σ**:

(a) $Y = 3B(1-2\sigma)$;

(b) $Y = 2\eta(1+\sigma)$;

(c) $\frac{1}{Y} = \frac{1}{9B} + \frac{1}{3\eta}$

(xiii) **Interatomic force constant**: $k = Yr_0$ (r_0 = equilibrium inter atomic separation)

KINETIC THEORY OF GASES

(i) **Boyle's law**: PV = constant or $P_1V_1 = P_2V_2$

(i) **Chare's law**: (V/T) = constant or $(V_1/T_1) = (V_2/T_2)$

(ii) **Pressure – temperature law**: $(P_1/T_1) = (P_2/T_2)$

(iii) **Avogadro's principle**: At constant temperature and pressure, Volume of gas,
$V \propto$ number of moles, μ
Where $\mu = N/Na$ [N = number of molecules in the sample
and N_A = Avogadro's number = 6.02×10^{23}/mole]

$= \frac{M_{sample}}{M}$ [M_{sample} = mass of gas sample and M = molecular weight]

(iv) **Kinetic Theory**:

(a) Momentum delivered to the wall perpendicular to the x–axis, $\Delta P = 2m\, v_x$

(b) Time taken between two successive collisions on the same wall by the same molecule: $\Delta t = (2L/v_x)$

(c) The frequency of collision: $\nu_{coll.} = (v_x/2L)$

(d) Total force exerted on the wall by collision of various molecules: $F = (MN/L) < v_x^2 >$

(e) The pressure on the wall : $P = \frac{mN}{V} < v_x^2 > = \frac{mN}{3V} < v^2 > = \frac{1}{3}\frac{mN}{V} v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$

(v) **RMS speed**:

(a) $v_{rms} = \sqrt{(v_1^2 + v_2^2 + \ldots + v_N^2/N)}$;

(b) $v_{rms} = \sqrt{(3P/\rho)}$;

(c) $v_{rms} = \sqrt{(3KT/m)}$;

(d) $v_{rms} = \sqrt{(3RT/M)}$; (e) $\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{M_2}{M_1}}$

(vi) **Kinetic interpretation of temperature**:

(a) (1/2) Mv^2_{rms} = (3/2) RT ;
(b) (1/2) mv^2_{rms} = (3/2) KT
(c) Kinetic energy of one molecule = (3/2) KT ;
(d) kinetic energy of one mole of gas = (3/2) RT
(e) Kinetic energy of one gram of gas (3/2) (RT/M)

(ix) Maxwell molecular speed distribution:

(a) $n(v) = 4\pi N \left(\frac{m}{2\pi KT}\right)^{3/2} v^2 e^{-mv^2/2KT}$

(b) The average speed: $\bar{v} = \sqrt{\frac{8KT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = 1.60\sqrt{\frac{RT}{M}}$

(c) The rms speed: $v_{rms} = \sqrt{\frac{3kT}{m}} = \frac{\sqrt{3RT}}{M} = 1.73\sqrt{\frac{RT}{M}}$

(d) The most probable speed: $v_p = \sqrt{\frac{2KT}{m}} = \sqrt{\frac{2RT}{M}} = 1.41\sqrt{\frac{RT}{M}}$

(e) Speed relations: (I) $v_p < \bar{v} < v_{rms}$
(II) $v_p : \bar{v} : v_{rms} = \sqrt{(2)} : \sqrt{(8/\pi)} : \sqrt{(3)}$ = 1.41 : 1.60 : 1.73

(x) Internal energy:
(a) $E_{internal}$ = (3/2)RT (for one mole)
(b) $E_{internal}$ = (3/2 μRT (for μ mole)
(c) Pressure exerted by a gas $P = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\overline{E}$

(xi) Degrees of freedom:
(a) Ideal gas: 3 (all translational)
(b) Monoatomic gas : 3 (all translational)
(c) Diatomic gas: 5 (three translational plus two rotational)
(d) Polyatomic gas (linear molecule e.g. CO_2) : 7 (three translational plus two rotational plus two vibrational)
(e) Polyatomic gas (non–linear molecule, e.g., NH_3, H_2O etc): 6 (three translational plus three rotational)
(f) Internal energy of a gas: $E_{internal}$ = (f/2) μRT. (where f = number of degrees of freedom)

(xii) Dalton's law: The pressure exerted by a mixture of perfect gases is the sum of the pressures exerted by the individual gases occupying the same volume alone i.e., $P = P_1 + P_2 + \ldots$.

(xiii) Van der Wall's gas equation:

(a) $\left(P + a\frac{\mu^2}{V^2}\right)(V - \mu b) = \mu RT$

(b) $\left(P + a\frac{\mu^2}{V_m^2}\right)(V_m - b) = RT$ (where $V_m = V/\mu$ = volume per mole);

(c) b = 30 cm^3/mole

(d) Critical values: $P_c = \frac{a}{27b^2}, V_C = 3b, T_C = \frac{8a}{27Rb}$;

(e) $\frac{P_C V_C}{RT_C} = \frac{3}{8} = 0.375$

(xiv) Mean free path: $\lambda = \frac{1}{\sqrt{2\pi d^2 \rho n}}$,

Where ρ_n = (N/V) = number of gas molecules per unit volume and
d = diameter of molecules of the gas

FLUID MECHANICS

(i) The viscous force between two layers of area A having velocity gradient (dv/dx) is given by: $F = -\eta A\,(dv/dx)$, where η is called coefficient of viscosity

(i) In SI system, η is measured I Poiseiulle ($P\ell$) $1P\ell = 1 Nsm^{-2}$ = 1 decapoise. In egs system, the unit of η is g/cm/sec and is called POISE

(ii) When a spherical body is allowed to fall through viscous medium, its velocity increases, till the sum of viscous drag and upthrust becomes equal to the weight of the body. After that the body moves with a constant velocity called terminal velocity.

(iii) According to STOKE's Law, the viscous drag on a spherical body moving in a fluid is given by: $F = 6\pi\eta r\,v$, where r is the radius and v is the velocity of the body.

(iv) The terminal velocity is given by: $v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

where ρ is the density of the material of the body and σ is the density of liquid

(v) Rate of flow of liquid through a capillary tube of radius r and length ℓ

$$V = \frac{\pi p r^4}{8\eta\ell} = \frac{p}{8\eta\ell/\pi r^4} = \frac{p}{R}$$

where p is the pressure difference between two ends of the capillary and R is the fluid resistance ($=8\,\eta\ell/\pi r^4$)

(vi) The matter which possess the property of flowing is called as FLUID (For example, gases and liquids)

(vii) Pressure exerted by a column of liquid of height h is : $P = h\rho g$ (ρ = density of the liquid)

(viii) Pressure at a point within the liquid, $P = P_0 + h\rho g$, where P_0 is atmospheric pressure and h is the depth of point w.r.t. free surface of liquid

(ix) Apparent weight of the body immersed in a liquid $Mg' = Mg - V\rho g$

(x) If W be the weight of a body and U be the upthrust force of the liquid on the body then
- (a) the body sinks in the liquid of W > U
- (b) the body floats just completely immersed if W = U
- (c) the body floats with a part immersed in t

(xi) $\frac{\text{Volume of immersed part of a solid}}{\text{total volume of solid}} = \frac{\text{density of solid}}{\text{density of solid}}$

(xii) Equation of Continuity: $a_1v_1 = a_2v_2$

(xiii) Bernouilli's theorem: $(P/\rho) + gh + \frac{1}{2}v^2$ = constant

(xiv) Accelerated fluid containers : $\tan\theta = \frac{a_x}{g}$

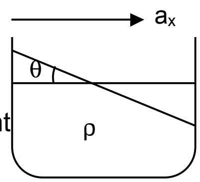

Fig. 4

(xv) Volume of liquid flowing per second through a tube: $R = a_1v_1 = a_2v_2\sqrt{\frac{2gh}{(a_1^2 - a_2^2)}}$

(xvi) Velocity of efflux of liquid from a hole:
$v = \sqrt{(2gh)}$, where h is the depth of a hole from the free surface of liquid

I do not ask to walk smooth paths, nor bear an easy load.
I pray for strength and fortitude to climb rock-strewn road.
Give me such courage I can scale the hardest peaks alone,
And transform every stumbling block into a stepping-stone.

– Gail Brook Burkett

HEAT AND THERMODYNAMICS

(i) $L_2 - L_1 = L_1\alpha(T_2 - T_1)$; $A_2 - A_1 = A_2\,\beta(T_2 - T_1)$; $V_2 - V_1 = V_1\gamma(T_2 - T_1)$
where, L_1, A_1, V_1 are the length, area and volume at temperature T_1; and L_2, A_2, V_2 are that at temperature T_2.α represents the coefficient of linear expansion, β the coefficient of superficial expansion and γ the coefficient of cubical expansion.

(ii) If d_t be the density at t^0C and d_0 be that at 0^0C, then: $d_t = d_0\,(1-\gamma\Delta T)$
(iii) $\alpha : \beta : \gamma = 1 : 2 : 3$
(iv) If γ_r, γ_a be the coefficients of real and apparent expansions of a liquid and γ_g be the coefficient of the cubical expansion for the containing vessel (say glass), then
$\gamma_r = \gamma_a + \gamma_g$

(v) The pressure of the gases varies with temperature as : $P_t = P_0\,(1+ \gamma\Delta T)$, where $\gamma = (1/273)$ per ^{0}C
(vi) If temperature on Celsius scale is C, that on Fahrenheit scale is F, on Kelvin scale is K, and on Reaumer scale is R, then

(a) $\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{R}{4}$ (b) $F = \frac{9}{5}C + 32$

(c) $C = \frac{5}{9}(F-32)$

(d) $K = C + 273$ (e) $K = \frac{5}{9}(F + 459.4)$

(vii) (a) Triple point of water = 273.16 K
(b) Absolute zero = 0 K = -273.15^0C
(c) For a gas thermometer, $T = (273.15)\,\frac{P}{P_{triple}}(\text{Kelvin})$
(d) For a resistance thermometer, $R_\theta = R_0\,[1+ \alpha\theta]$

(viii) If mechanical work W produces the same temperature change as heat H, then we can write: W = JH, where J is called mechanical equivalent of heat
(ix) The heat absorbed or given out by a body of mass m, when the temperature changes by ΔT is: $\Delta Q = mc\Delta T$, where c is a constant for a substance, called as SPECIFIC HEAT.
(x) HEAT CAPACITY of a body of mass m is defined as : $\Delta Q = mc$
(xi) WATER EQUIVALENT of a body is numerically equal to the product of its mass and specific heat i.e., W = mc
(xii) When the state of matter changes, the heat absorbed or evolved is given by: Q = mL, where L is called LATENT HEAT
(xiii) In case of gases, there are two types of specific heats i.e., c_p and c_v [c_p = specific heat at constant pressure and C_v = specific heat at constant volume]. Molar specific heats of a gas are: $C_p = Mc_p$ and $C_v = Mc_v$, where M = molecular weight of the gas.
(xiv) $C_p > C_v$ and according to Mayer's formula $C_p - C_v = R$

(xv) For all thermodynamic processes, equation of state for an ideal gas: $PV = \mu RT$
(a) For ISOBARIC process: P = Constant ; $\frac{V}{T}$ =Constant
(b) For ISOCHORIC (Isometric) process: V = Constant; $\frac{P}{T}$ =Constant
(c) For ISOTHERMAL process T = Constant ; PV= Constant
(d) For ADIABATIC process: PV^γ = Constant ; $TV^{\gamma-1}$=Constant
and $P^{(1-\gamma)}\,T^\gamma$ = Constant

(xvi) Slope on PV diagram

(a) For isobaric process: zero
(b) For isochoric process: infinite
(c) For isothermal process: slope = –(P/V)
(d) For adiabatic process: slope = –γ(P/V)
(e) Slope of adiabatic curve > slope of isothermal curve.

(xvii) Work done

(a) For isobaric process: $W = P(V_2 - V_1)$
(b) For isochoric process: $W = 0$
(c) For isothermal process: $W = \mu RT \log_e (V_2/V_1)$

$$\mu RT \times 2.303 \times \log_{10} (V_2/V_1)$$
$$P_1V_1 \times 2.303 \times \log_{10} (V_2/V_1)$$
$$\mu RT \times 2.303 \times \log_{10} (P_1/P_2)$$

(d) For adiabatic process: $W = \frac{\mu R (T_1 - T_2)}{(\gamma - 1)} = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)}$

(e) In expansion from same initial state to same final volume

$$W_{adiabatic} < W_{isothermal} < W_{isobaric}$$

(f) In compression from same initial state to same final volume:

$$W_{adiabatic} < W_{isothermal} < W_{isobaric}$$

(xviii) **Heat added or removed**:

(a) For isobaric process: $Q = \mu C_p \Delta T$
(b) For isochoric process = $Q = \mu C_v \Delta T$
(c) For isothermal process = $Q = W = \mu Rt \log_e (V_2/V_1)$
(d) For adiabatic process: $Q = 0$

(xix) Change in internal energy

(a) For isobaric process = $\Delta U = \mu C_v \Delta T$
(b) For isochoric process = $\Delta U = \mu C_v \Delta T$
(c) For isothermal process = $\Delta U = 0$
(d) For adiabatic process: $\Delta U = -W = \frac{\mu R (T_2 - T_1)}{(\gamma - 1)}$

(xx) Elasticities of gases

(a) Isothermal bulk modulus = $B_I = P$
(b) Adiabatic bulk modulus $B_A = \gamma P$

(xxi) For a CYCLIC process, work done ΔW = area enclosed in the cycle on PV diagram. Further, $\Delta U = 0$ (as state of the system remains unchanged)
So, $\Delta Q = \Delta W$

(xxii) Internal energy and specific heats of an ideal gas (Monoatomic gas)

(a) $U = \frac{3}{2} RT$ (for one mole);

(b) $U = \frac{3}{2} \mu RT$ (for μ moles)

(c) $\Delta U = \frac{3}{2} \mu R \Delta T$ (for μ moles);

(d) $C_v = \frac{1}{\mu}\left(\frac{\Delta U}{\Delta T}\right) = \frac{3}{2} R$

(e) $C_p = Cv + R = \frac{3}{2}R + R = \frac{5}{2}R$

(f) $\gamma = \left(\frac{C_p}{C_v}\right) = \left(\frac{5}{2}R \Big/ \frac{3}{2}R\right) = \frac{5}{3} = 1.67$

(xxiii) Internal energy and specific heats of a diatomic gas

(a) $U = \frac{5}{2}\mu RT$ (for μ moles);

(b) $\Delta U = \frac{5}{2}\mu R\Delta T$ (for μ moles)

(c) $C_v = \frac{1}{\mu}\left(\frac{\Delta U}{\Delta T}\right) = \frac{5}{2}R;$

(d) $C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R$

(e) $\gamma = \left(\frac{C_p}{C_v}\right) = \left(\frac{7R}{2} \Big/ \frac{5R}{2}\right) = \frac{7}{5} = 1.4$

(xxiv) **Mixture of gases**: $\mu = \mu_1 + \mu_2$

$$M = \frac{\mu_1 M_1 + \mu_2 + M_2}{\mu_1 + \mu_2} = \frac{N_1 m_1 + N_2 m_2}{N_1 + N_2}$$

$$C_v = \frac{\mu_1 C_{v_1} + \mu_2 C_{v_2}}{\mu_1 + \mu_2} \quad \text{and} \quad C_p = \frac{\mu_1 C_{p_1} + \mu_2 C_{p_2}}{\mu_1 + \mu_2}$$

(xxv) First law of thermodynamics

(a) $\Delta Q = \Delta U + \Delta W$ or $\Delta U = \Delta Q - \Delta W$

(b) Both ΔQ, ΔW depends on path, but ΔU does not depend on the path

(c) For isothermal process: $\Delta Q = \Delta W = \mu RT \log | V_2/V_1|$, $\Delta U = 0$, T = Constant, PV = Constant and $C_{iso} = \pm\infty$

(d) For adiabatic process: $\Delta W = \frac{\mu R(T_2 - T_1)}{(1-\gamma)}$, $\Delta Q = 0$, $\Delta U = \mu C_v (T_2 - T_1)$, Q = 0,

PV^γ = constant, $C_{ad} = 0$ and $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

(where f is the degree of freedom)

(e) For isochoric process: $\Delta W = 0$, $\Delta Q = \Delta U = \mu C_v\Delta T$, V = constant, and $C_v = (R/\gamma-1)$

(f) For isobaric process: $\Delta Q = \mu C_p\Delta T$, $\Delta U = \mu C_v\Delta T$., $\Delta W = \mu R\Delta T$, P = constant and $C_p = (\gamma R/\gamma-1)$

(g) For cyclic process: $\Delta U = 0$, $\Delta Q = \Delta W$

(h) For free expansion: $\Delta U = 0$, $\Delta Q = 0$, $\Delta W = 0$

(i) For polytropic process: $\Delta W = [\mu R(T_2-T_1)/1-n]$, $\Delta Q = \mu C (T_2-T_1)$, PV^n = constant and

$$C = \frac{R}{\gamma-1} + \frac{R}{1-n}$$

(xxvi) Second law of thermodynamics

(a) There are no perfect engines

(b) There are no perfect refrigerators

(c) Efficiency of carnot engine: $\eta = 1 - \frac{Q_2}{Q_1}, \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

(d) Coefficient of performance of a refrigerator:

$$\beta = \frac{\text{Heat absorbed from cold reservoir}}{\text{Work done on refrigerator}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

For a perfect refrigerator, $W = 0$ or $Q_1 = Q_2$ or $\beta = \infty$

(xxvii) **The amount of heat transmitted is given by**: $Q = -KA \frac{\Delta\theta}{\Delta x} t$, where K is coefficient of thermal conductivity, A is the area of cross section, $\Delta\theta$ is the difference in temperature, t is the time of heat flow and Δx is separation between two ends

(xxviii) **Thermal resistance of a conductor of length** $d = R_{Th} = \frac{d}{KA}$

(xxix) **Flow of heat through a composite conductor**:

(a) Temperature of interface, $\theta = \frac{(K_1\theta_1/d_1) + (K_2\theta_2/d_2)}{(K_1/d_1) + (K_2/d_2)}$

(b) Rate of flow of heat through the composite conductor: $H = \frac{Q}{t} = \frac{A(\theta_1 - \theta_2)}{(d_1/K_1) + (d_2/K_2)}$

(c) Thermal resistance of the composite conductor

$$R_{TH} = \frac{d_1}{K_1 A} + \frac{d_2}{K_2 A} = (R_{Th})_1 + (R_{Th})_2$$

(d) Equivalent thermal conductivity, $K = \frac{d_1 + d_2}{(d_1/K_1) + (d_2/K_2)}$

(xxx) (a) Radiation absorption coefficient: $a = Q_0/Q_0$

(b) Reflection coefficient: $r = Q_r/Q_0$

(c) Transmission coefficient: $t = Q_t/Q_0$

(d) Emissive power: e or $E = Q/A.t$ [t = time]

(e) Spectral emissive power: $e_\lambda = \frac{Q}{At(d\lambda)}$ and $e = \int_0^\infty e_\lambda d\lambda$

(f) Emissivity: $\varepsilon = e/E$; $0 \le \varepsilon \le 1$

(g) Absorptive power: $a = Q_a/Q_0$

(h) Kirchhoffs law: $(e_\lambda/a_\lambda)1 = (e_\lambda/a_\lambda)_B = \dots\dots\dots = E_\lambda$

(i) Stefan's law: (a) $E = \sigma T^4$ (where $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$)

For a black body: $E = \sigma(T^4 - T_0^4)$

For a body: $e = \varepsilon\sigma(T^4 - T_0^4)$

(j) Rate of loss of heat: $-\frac{dQ}{dt} = \varepsilon A\sigma(\theta^4 - \theta_0^4)$

For spherical objects: $\frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{r_1^2}{r_2^2}$

(k) Rate of fall of temperature: $\frac{d\theta}{dt} = \frac{\varepsilon A\sigma}{ms}(\theta^4 - \theta_0^4) = \frac{\varepsilon A\sigma}{V\rho s}(\theta^4 - \theta_0^4)$

$\therefore$ $\frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{A_1}{A_2} \times \frac{V_2}{V_1} = \frac{r_2}{r_1}$

(For spherical bodies)

(l) Newton's law of cooling: $\frac{d\theta}{dt} = -K(\theta-\theta_0)$ or $(\theta-\theta_0) \; \alpha \; e^{-KT}$

(m) Wein's displacement law: $\lambda_m T = b$ (where $b = 2.9 \times 10^{-3}$ m – K)

(n) Wein's radiation law: $E_\lambda d\lambda = \left(\frac{A}{\lambda^5}\right) f(\lambda T)\, d\lambda = \left(\frac{A}{\lambda^5}\right) e^{-a/\lambda T} d\lambda$

(o) Solar Constant: $S = \left(\frac{R_S}{R_{ES}}\right)^2 \sigma T^4$ or $T = \left(\frac{S}{\sigma}\right)^{1/4} \left(\frac{R_{ES}}{R_S}\right)^{1/2}$

WAVES

1. **Velocity**: $v = n\lambda$ and $n = (1/T)$

2. **Velocity of transverse waves in a string**: $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\pi r^2 d}}$

3. **Velocity of longitudinal waves**:
   (a) In rods: $v = \sqrt{(Y/\rho)}$ (Y – Young's modulus, ρ = density)
   (b) In liquids: $v = \sqrt{(B/\rho)}$ (B = Bulk modulus)
   (c) In gases: $v = \sqrt{(\gamma P/\rho)}$ (Laplace formula)

4. **Effect of temperature**:
   (a) $v = v_0\sqrt{(T/273)}$ or $v = v_0 + 0.61t$
   (b) $(v_{sound}/v_{rms}) = \sqrt{(\gamma/3)}$

5. **Wave equation**: (a) $y = a \sin \frac{2\pi}{\lambda} (vt - x)$

   (b) $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

   (c) $y = a \sin (\omega t - kx)$, where wave velocity $v = \frac{\omega}{k} = n\lambda$

6. **Particle velocity**: (a) $v_{particle} = (\partial y/\partial t)$
   (b) maximum particle velocity, $(v_{particle})max = \omega a$

7. **Strain in medium** (a) strain $= -(\partial y/\partial x) = ka \cos (\omega t - kx)$
   (b) Maximum strain $= (\partial y/dx)_{max} = ka$
   (c) $(v_{particle}/\text{strain}) = (\omega/k)$ = wave velocity
   i.e., $v_{particle}$ = wave velocity x strain in the medium

8. **Wave equation**: $\frac{\partial^2 y}{\partial t^2} = v^2 \left(\frac{\partial^2 y}{\partial x^2}\right)$

9. **Intensity of sound waves**:
   (a) $I = (E/At)$
   (b) If ρ is the density of the medium; v the velocity of the wave; n the frequency and a the amplitude then $I = 2\pi^2 \rho v n^2 a^2$ i.e. $I \propto n^2a^2$
   (c) Intensity level is decibel: β 10 log (I/I_0). Where, I_0 =Threshold of hearing = 10^{-12} Watt/m^2

10. **Principle of superposition**: $y = y_1 + y_2$

11. **Resultant amplitude**: $a = \sqrt{(a_1^2 + a_2^2 + 2a_1a^2 \cos \phi)}$

12. **Resultant intensity**: $I = I_1 + I_2 + 2\sqrt{(I_1I_2 \cos \phi)}$
    (a) For constructive interference: $\phi = 2n\pi$, $a_{max} = a_1 + a_2$ and $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$
    (b) For destructive interference: $\phi = (2n-1)\pi$, $a_{min} = a_2 - a_2$ and $I_{min} = (\sqrt{I_1} = \sqrt{I_2})^2$

13. (a) Beat frequency $= n_1 - n_2$ and beat period $T = (T_1T_2/T_2 - T_1)$
    (b) If there are N forks in successive order each giving x beat/sec with nearest neighbour, then $n_{last} = n_{first} + (N-1)x$

14. **Stationary waves**: The equation of stationary wave,
    (a) When the wave is reflected from a free boundary, is:

$$y = + 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T} = 2a \cos kx \sin \omega t$$

(b) When the wave is reflected from a rigid boundary, is:

$$Y + -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi t}{T} = -2a \sin kx \cos \omega t$$

15. **Vibrations of a stretched string**:

(a) For fundamental tone: $n_1 = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$

(b) For p th harmonic : $n_p = \frac{p}{\lambda}\sqrt{\frac{T}{m}}$

(c) The ratio of successive harmonic frequencies: $n_1 : n_2 : n_3 :$…….. = 1 : 2 : 3 : ……

(d) Sonometer: $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ $(m = \pi r^2 d)$

(e) Melde's experiment: (i) Transverse mode: $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

(ii) Longitudinal mode: $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

16. **Vibrations of closed organ pipe**

(a) For fundamental tone: $n_1 = \left(\frac{v}{4L}\right)$

(b) For first overtone (third harmonic): $n_2 = 3n_1$

(c) Only odd harmonics are found in the vibrations of a closed organ pipe and $n_1 : n_2 : n_3 :$ …..=1 : 3 : 5 : ……

17. **Vibrations of open organ pipe**:

(a) For fundamental tone: $n_1 = (v/2L)$

(b) For first overtone (second harmonic) : $n_2 = 2n_1$

(c) Both even and odd harmonics are found in the vibrations of an open organ pipe and $n_1 : n_2 : n_3 :$ ……=1 : 2 : 3 : …….

18. **End correction**: (a) Closed pipe : $L = L_{pipe} + 0.3d$

(b) Open pipe: $L = L_{pipe} + 0.6\,d$

where d = diameter = 2r

19. **Resonance column**: (a) $\ell_1 + e = \frac{\lambda}{4}$; (b) $\ell_2 + e = \frac{3\lambda}{4}$

(c) $e = \frac{\ell_2 - 3\ell_1}{2}$; (d) $n = \frac{v}{2(\ell_2 - \ell_1)}$ or $\lambda = 2(\ell_2 - \ell_1)$

20. **Kundt's tube**: $\frac{v_{air}}{v_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}}$

21. **Longitudinal vibration of rods**

(a) Both ends open and clamped in middle:

(i) Fundamental frequency, $n_1 = (v/2\ell)$

(ii) Frequency of first overtone, $n_2 = 3n_1$

(iii)Ratio of frequencies, $n_1 : n_2 : n_3 :$ …… = 1 3: 5 : …..

(b) One end clamped

(i) Fundamental frequency, $n_1 = (v/4\ell)$

(ii) Frequency of first overtone, $n_2 = 3n_1$

(iii) Ratio of frequencies, $n_1 : n_2 : n_3 : ......= 1 : 3 : 5 : ....$

22. **Frequency of a turning fork**: $n \propto \frac{t}{\ell_2}\sqrt{\frac{E}{\rho}}$

Where t = thickness, ℓ = length of prong, E = Elastic constant and ρ = density

23. **Doppler Effect for Sound**

(a) Observer stationary and source moving:

(i) Source approaching: $n' = \frac{v}{v - v_s} \times n$ and $\lambda' = \frac{v - v_s}{v} \times \lambda$

(ii) Source receding: $n' = \frac{v}{v + v_s} \times n$ and $\lambda' = \frac{v + v_s}{v} \times \lambda$

(b) Source stationary and observer moving:

(i) Observer approaching the source: $n' = \frac{v + v_0}{v} \times n$ and $\lambda' = \lambda$

(ii) Observer receding away from source: $n' = \frac{v - v_0}{v} \times n$ and $\lambda' = \lambda$

(c) Source and observer both moving:

(i) S and O moving towards each other: $n' = \frac{v + v_0}{v - v_s} \times n$

(ii) S and O moving away from each other: $n' = \frac{v - v_0}{v + v_s} \times n$

(iii) S and O in same direction, S behind O : $n' = \frac{v - v_0}{v - v_s} \times n$

(iv) S and O in same direction, S ahead of O: $n' = \frac{v + v_0}{v + v_s} \times n$

(d) Effect of motion of medium: $n' = \frac{v \pm v_m \pm v_0}{v \pm v_m \pm v_s}$

(e) Change in frequency: (i) Moving source passes a stationary observer: $\Delta n = \frac{2vv_s}{v^2 - v_s^2} \times n$

For $v_s << v$, Δ, $= \frac{2v_s}{v} \times n$

(ii) Moving observer passes a stationary source: $\Delta n = \frac{2v_0}{v} \times n$

(f) Source moving towards or away from hill or wall

(i) Source moving towards wall

(a) Observer between source and wall

$n' = \frac{v}{v - v_s} \times n$ (for direct waves)

$n' = \frac{v}{v - v_s} \times n$ (for reflected waves)

(b) Source between observer and wall

$$n' = \frac{v}{v + v_s} \times n \quad \text{(for direct waves)}$$

$$n' = \frac{v}{v - v_s} \times n \quad \text{(for reflected waves)}$$

(ii) Source moving away from wall

(a) Observer between source and wall

$$n' = \frac{v}{v + v_s} \times n \quad \text{(for direct waves)}$$

$$n' = \frac{v}{v + v_s} \times n \quad \text{(for reflected waves)}$$

(b) Source between observer and wall

$$n' = \frac{v}{v - v_s} \times n \quad \text{(for direct waves)}$$

$$n' = \frac{v}{v + v_s} \times n \quad \text{(for reflected waves)}$$

(g) Moving Target:

(i) S and O stationary at the same place and target approaching with speed u

$$n' = \left(\frac{v+u}{v-u}\right) \times n \quad \text{or} \quad n' = \left(1 + \frac{2u}{v}\right) \times n \quad \text{(for } u << v)$$

(ii) S and O stationary at the same place and target receding with speed u

$$n' = \left(\frac{v-u}{v+u}\right) \times n \quad \text{or} \quad n' = \left(1 - \frac{2u}{v}\right) \times n \quad \text{(for } u << v)$$

(h) SONAR: $n' = \frac{v \pm v_{sub}}{v \pm v_{sub}} \times n \cong \left(1 \pm \frac{2\,v_{sub}}{v}\right) \times n$

(upper sign for approaching submarine while lower sign for receding submarine)

(i) Transverse Doppler effect: There is no transverse Doppler effect in sound. For velocity component $v_s \cos\theta$

$n' = \frac{v}{v \pm v_s \cos\theta} \times n$ (– sign for approaching and + sign for receding)

24. Doppler Effect for light

(a) Red shift (when light source is moving away):

$$n' = \sqrt{\frac{1 - v/c}{1 + v/c}} \times n \quad \text{or} \quad \lambda' = \sqrt{\frac{1 + v/c}{1 - v/c}} \times \lambda$$

For $v << c$, $\Delta n = -\left(\frac{v}{c}\right) \times n$ or $\Delta\lambda' = \left(\frac{v}{c}\right) \times \lambda$

(b) Blue shift (when light source is approaching)

$$n' = \sqrt{\frac{1 + v/c}{1 - v/c}} \times n \quad \text{or} \quad \lambda' = \sqrt{\frac{1 - v/c}{1 + v/c}} \times \lambda$$

For $v << c$, $\Delta n = \left(\frac{v}{c}\right) n$ or $\Delta\lambda' = -\left(\frac{v}{c}\right)\lambda$

(c) Doppler Broadening = $2\Delta\lambda = 2\left(\frac{v}{c}\right)\lambda$

(d) Transverse Doppler effect:

For light, $n' = \sqrt{1-\frac{v^2}{c^2}} \text{ x } n = \left(1-\frac{1}{2}\frac{v^2}{c^2}\right) \text{ x } n$ (for v << c)

(e) RADAR: $\Delta_n = \left(\frac{2v}{c}\right)n$

STUDY TIPS

- **Combination of Subjects**

  Study a combination of subjects during a day i. e. after studying 2–3 hrs of mathematics shift to any theoretical subject for 2 horrs. When we study a subject like math, a particular part of the brain is working more than rest of the brain. When we shift to a theoretical subject, practically the other part of the brain would become active and the part studying maths will go for rest.

- **Revision**

  Always refresh your memory by revising the matter learned. At the end of the day you must revise whatever you've learnt during that day (or revise the previous days work before starting studies the next day). On an average brain is able to retain the newly learned information 80% only for 12 hours, after that the forgetting cycle begins. After this revision, now the brain is able to hold the matter for 7 days. So next revision should be after 7 days (sundays could be kept for just revision). This ways you will get rid of the problem of forgetting what you study and save a lot of time in restudying that topic.

- **Use All Your Senses**

  Whatever you read, try to convert that into picture and visualize it. *Our eye memory is many times stronger than our ear memory* since the nerves connecting brain to eye are many times stronger than nerves connecting brain to ear. So instead of trying to mug up by repeating it loudly try to see it while reapeating (loudly or in your mind). This is applicable in theoritical subjects. Try to use all your senses while learning a subject matter. On an average we remember 25% of what we read, 35% of what we hear, 50% of what we say, 75% of what we see, 95% of what we read, hear, say and see.

- **Breathing and Relaxation**

  Take special care of your breathing. Deep breaths are very important for relaxing your mind and hence in your concentration. Pranayam can do wonders to your concentration, relaxation and sharpening your mined (by supplying oxygen to it). Aerobic exercises like skipping, jogging, swimming and cycling are also very helpful.