Class XI: Physics
Chapter 3: Motion in Plane
Chapter Notes

Key Learnings

1. Scalar quantities are quantities with magnitudes only. Examples are distance, speed, mass and temperature.

2. Vector quantities are quantities with magnitude and direction both. Examples are displacement, velocity and acceleration. They obey special rules of vector algebra.

3. A vector A multiplied by a real number λ is also a vector, whose magnitude dependent upon whether λ is positive or negative.

4. Two vectors A and B may be added graphically using head – to – tail method or parallelogram method.

5. Vector addition is commutative:

   A + B – B + A

   It also obeys the associative law:

   (A + B) + C = A + (B+C)

6. A null or zero vector is a vector with zero magnitude. Since the magnitude is zero, we don't have to specify its direction. It has sthe properties:

$$A + O = A$$

$$\lambda O = O$$

$$OA = O$$

7. The subtraction of vector B from A is defined as the sum of A and –B:

   A – B = A + (-B)

8. A vector A can be resolved into component along two given vectors a and b lying in the same plane:

A = λa + μb

Where λ and μ are real numbers.

9. A unit vector associated with a vector A has magnitude one and is along the vector A:

$$\hat{n} = \frac{A}{|A|}$$

The unit vectors $\hat{i}, \hat{j}, \hat{k}$ are vectors of unit magnitude and point in the direction of the x, y, and z – axes, respectively in a right – handed coordinate system.

11. Two vectors can be added geometrically by placing them head to tail. The vector connecting the tail of the first to the head of the second is the vector sum or resultant vector.

12. Vector R can be resolved into perpendicular components given as Rx and Ry along x and y axis respectively.

$$R_x = R\cos\theta \text{ and } R_y = R\sin\theta$$

An efficient method for adding vectors is using method of components.

13. Unit vectors i, j, and k have magnitudes of unity and are directed in the positive direction of the x, y and z axes.

14. The position vector of particle at that instant is a vector that goes from the origin of the coordinate system to that point P.

15. The displacement vector is equal to the final position vector minus the initial position vector.

16. Average velocity vector is equal to change in position vector divided by the corresponding time interval.

17. Instantaneous velocity or simply velocity of a particle is along the tangent to the particle's path at each instant

18. Average acceleration is a vector quantity in the same direction as the velocity vector.

19. Projectile is an object on which the only force acting is gravity.

20. The projectile motion can be thought of as two separate simultaneously occurring components of motion along the vertical and horizontal directions.

21. During a projectile's flight its horizontal acceleration is zero and vertical acceleration is $-9.8 m/s^2$..

22. The trajectory of particle in projectile motion is parabolic.

23. When a body P moves relative to a body B and B moves relative to A, then velocity of P relative to A is velocity of P relative to B + velocity of P relative to A.

$$\vec{V}_{P/A} = \vec{V}_{P/B} + \vec{V}_{B/A}$$

24. $\vec{V}_{A/B} = -\vec{V}_{B/A}$

25. When an object follows a circular path at constant speed, the motion of the object is called uniform circular motion. The magnitude of its acceleration is $a_c = v^2 / R$. The direction of a_c is always towards the centre of the circle.

26. The angular speed w, is the rate of change of angular distance. It is related to velocity v by v = wR. The acceleration is $a_c = w^2 R$.

27. If T is the time period of revolution of the object in circular motion and v is the frequency, we have w = 2π vR, $a_c = 4\pi^2 v^2 R$

Top Formulae:

Projectile Motion

Thrown at an angle with horizontal

(a) $y = x\tan\theta - \frac{1}{2}.g.\left[\frac{x}{u\cos\theta}\right]^2$

$\overrightarrow{U_x} = u\cos\theta\,\hat{i} \qquad a_x = 0$

$\vec{U}_y = u\sin\theta\,\hat{j} \qquad a_x = -g\,\hat{j}$

Or $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

(b) Time to reach max, height $t = \frac{u\sin\theta}{g} = \frac{u_y}{a_y}$

(c) Time of flight $T = \frac{2u\sin\theta}{g} = \frac{2u_y}{a_y}$

(d) Horizontal range $R = \frac{u^2\sin 2\theta}{g} = u_x \times T$

(e) Max. height $H_{max} = \frac{u^2\sin 2\theta}{2g} = \frac{u_y^2}{2a_y}$

(f) Horizontal velocity at any time

$v_x = u\cos\theta$

(g) Vertical component of velocity at any time

$v_y = u\sin\theta - gt$

(h) Resultant velocity $\vec{v} = v_x\hat{i} + v_y\hat{j}$

$\vec{v} = u\cos\theta\,\hat{j} + (u\sin\theta - gt)\,\hat{j}$

$v = |\vec{v}| = \sqrt{u^2 + g^2t^2 = 2ugt\sin\theta}$

And $\tan\alpha = \frac{v_y}{v_x}$

General Result

For max. range θ = 45°

$R_{max} = \frac{u^2}{g}$

And $H_{max} = \frac{R_{max}}{4}$, at $\theta = 45^\circ$ and initial velocity u

$= \frac{R_{max}}{2}$; at $\theta = 90^\circ$ and initial velocity u

(i) Change in momentum

(ii) For completer motion = $-2\ m\ u \sin\theta$

(iii) at highest point = $-m\ u \sin\theta\ \hat{j}$

Projectile thrown parallel to the horizontal

(a) Equation $y = -\frac{1}{2} g \frac{x^2}{u^2}$

$u_x = u$ $\quad v_x = u$

$u_y = 0$ $\quad v_y = gt$ (down ward)

$= -gt$ (upward)

(b) velocity at any time

$v = \sqrt{u^2 + g^2t^2}$

$\tan\alpha = \frac{v_y}{v_x}$

(c) Displacement $S = x\,\hat{i} + y\,\hat{j} = ut\,\hat{i} + \frac{1}{2}gt^2\,\hat{j}$

(d) Time of Flight $T = \sqrt{\frac{2h}{g}}$

(e) Horizontal range $R = u\sqrt{\frac{2h}{g}}$

Projectile thrown from an inclined plane

$\vec{a}_x = -g\sin\theta_0\ \hat{i}$

$\vec{a}_y = -g\cos\theta_0\ \hat{j}$

$\vec{u}_x = u\cos(\theta - \theta_0)\ \hat{i}$

$\vec{u}_y = u\sin(\theta - \theta_0)\ \hat{j}$

(a) Time of flight $T = \frac{2u_y}{a_y} = \frac{2u\sin(\theta - \theta_0)}{g\cos\theta_0}$

$$R = u\cos(\theta - \theta_0)T - \frac{1}{2}g\sin\theta_0.T^2$$

$$R = \frac{2u^2\sin(\theta - \theta_0)\cos\theta}{g\cos^2\theta_0}$$

$$R = \frac{2u^2\sin(\theta - \theta_0)\cos\theta}{g\cos^2\theta_0}$$

Important for R_{max} = $\theta = \frac{\pi}{4} + \frac{\theta_0}{2}$ and $R_{max} = \frac{u^2}{g(1+\sin\theta_0)}$

Circular Motion

(a) angle (in radius) $= \frac{ac}{radius}$

Or $\Delta\theta = \frac{\Delta S}{r}$ π rad. = 180°

(b) Angular velocity ($\vec{\omega}$)

1. Instantaneous $\omega = \frac{d\theta}{dt}$

2. Average $\vec{\omega}_{av} = \frac{\text{total angular displacement}}{\text{total time taken}} = \frac{\Delta\theta}{\Delta t}$

If $v \rightarrow$ linear velocity

$\alpha \rightarrow$ angular acceleration

$a \rightarrow$ linear acceleration

(c) $v = r\,\omega$ In vector form $\vec{v} = \vec{\omega} \times \vec{r}$

(d) $\alpha = \frac{d\vec{\omega}}{dt}$

(e) $a = \alpha\, r$ and $\vec{a} = \vec{\alpha} \times \vec{r}$

Newtons equation in circular motion

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

Centripetal Force

$F_c = \frac{mv^2}{r} = m\omega^2 r$

$= m\,\omega\,v$

$a_c = \frac{v^2}{r}$ in vector $\vec{F}_c = m(\vec{v} \times \vec{\omega})$

Total Acceleration

$a_T = \sqrt{a_t^2 + a_c^2}$ $a_T \rightarrow$ Tangential acceleration

$a_c \rightarrow$ Centripetal acceleration

Motion In Horizontal Circle

$T \cos\theta = mg$

$T \sin\theta = mv^2 / r$ $\tan\theta = \frac{v^2}{rg}$

$$T = mg\sqrt{1 + \frac{v^4}{r^2g^2}}$$

The time period of revolution

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{\ell \cos\theta}{g}}$$

Banking of Tracks

$\tan\theta = \frac{v^2}{rg}$, on frictionless road, banked by θ

Maximum speed for skidding, on circular un-banked road

$v_{max} = \sqrt{\mu rg}$