Class XI: Physics
Chapter 7: Systems of Particles and Rotational Motion
Chapter Notes

Key Learnings

1. Rigid body is a solid body of finite size in which deformation is negligible under the effect of deforming forces.

2. A rigid body is one for which the distances between different particles of the body do not change.

3. Centre of mass (COM) of rigid body is the point in or near an object at which the whole mass of the object may be considered to be concentrated.

4. A rigid object can be substituted with a single particle with mass equal to the total mass of the system located at the COM of the rigid object.

5. In pure translational motion all particles of the body move with the same velocities in the same direction.

6. In pure translation, every particle of the body moves with the same velocity at any instant of time.

7. In rotational motion each particle of the body moves along the circular path in a plane perpendicular to the axis of rotation.

8. In rotation about a fixed axis, every particle of the rigid body moves in a circle with same angular velocity at any instant of time.

9. Irrespective of where the object is struck, the COM always moves in translational motion.

10. Motion of the COM is the resultant of the motions of all the constituent particles of a system.

11. Velocity of the centre of mass of a system of particles is given by $\vec{V} = \frac{\vec{P}}{M}$, where $\vec{P}$ is the linear momentum of the system.

12. The translational motion of the centre of mass of a system is, as if, all the mass of the system is concentrated at this point and all the external forces act at this point.

If the net external force on the system is zero, then the total linear momentum of the system is constant and the center of mass moves at a constant velocity.

13. Torque is the rotational analogue of force in translational motion.

14. The torque or moment of force on a system of n particles about the origin is the cross product of radius vectors and force acting on the particles.

$$\vec{\tau} = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i$$

15. Angular velocity in rotational motion is analogous to linear velocity in linear motion.

16. Conditions for equilibrium:

i. Resultant of all the external forces must be zero. Resultant of all the external torques must be zero.

ii. Centre of gravity is the location in the extended body where we can assume the whole weight of the body to be concentrated.

17. When a body acted by gravity is supported or balanced at a single point, the centre of gravity is always at and directly above or below the point of suspension.

18. The moment of inertia of a rigid body about an axis is defined by the formula $I = \sum m_i r_i^2$ where r_i is the perpendicular distance of the ith point of the body from the axis. The kinetic energy of rotation is $K = \frac{1}{2} I\omega^2$

19. Theorem of perpendicular axis:

It states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

$$I = I_x + I_y$$

Here

I_x : Moment of inertia about x axis in the plane of the lamina.

I_y : Moment of inertia about y axis in the plane of the lamina.

20. Theorem of parallel Axes:

This theorem states that the moment of inertia of a body about any axis is equal to its moment of inertia I_{cm} about a parallel axis through its center of mass, plus the product of the mass M of the body and the square of the distanced between the two axes.

$$I_p = I_{cm} + Md^2$$

21. Work done on the rigid body by the external torque is equal to the change in its kinetic energy.

22. Pure Rolling implies rolling without slipping which occurs when there is no relative motion at the point of contact where the rolling object touches the ground.

23. For a a rolling wheel of radius r which is accelerating, the acceleration of centre of mass -

$$a_{cm} = R\alpha$$

24. Law of conservation of angular momentum: If the net resultant external torque acting on an isolated system is zero, then total angular momentum L of system should be conserved.

25. The relation between the arc length S covered by a particle on a rotating rigid body at a distance r from the axis and the displacement theta in radians is given by S=r θ.

Top Formulae

1. The position vector of COM of a system:

$$\vec{R} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + ...}{m_1 + m_2 + m_3 + ...}$$

2. The coordinates of COM

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...},\ y = \frac{m_1y_1 + m_2y_2 + m_3y_3 + ...}{m_1 + m_2 + m_3 + ...},\ z = \frac{m_1z_1 + m_2z_2 + m_3z_3 + ...}{m_1 + m_2 + m_3 + ...}$$

3. Velocity of COM of a system of two two particles

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_1}{m_1 + m_2}$$

4. Equations of rotational motion

i. $\omega_2 = \omega_1 + \alpha t$

ii. $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

iii. $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

5. Centripetal acceleration = $\frac{v^2}{r} = r\omega^2$

6. Linear acceleration $a = r\alpha$

7. Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

8. Torque $\vec{\tau} = \vec{r} \times \vec{F}$

9. Kinetic energy of rotation = $\frac{1}{2} I\omega^2$

10. Kinetic energy of translation = $\frac{1}{2} mv^2$

11. Total K. E. = $\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

12. Angular momentum L = $I\omega$

13. Torque $\tau = I\alpha$

14. Relation between torque and angular momentum $\vec{\tau} = \frac{d\vec{L}}{dt}$

15. Moment of inertia in terms of radius of gyration I = $\sum_{i=1}^{i=n} m_i r_i^2 = MK^2$

16. Moment of inertia of a uniform circular ring about an axis passing through the centre and perpendicular to the plane of the ring, I = MR^2

17. For a uniform circular disc, I= $\frac{1}{2} MR^2$

18. For a thin uniform rod = $\frac{1}{12} M\ell^2$

19. For a hollow cylinder about its axis = MR^2

20. For a solid cylinder about its axis = $\frac{1}{2} MR^2$

21. For a hollow sphere about its diameter = $\frac{2}{3} MR^2$

22. For a solid sphere about its diameter = $\frac{2}{5} MR^2$

23. Power of a torque

24. Coefficient of friction for rolling of solid cylinder without slipping down the rough inclined plane $\mu = \frac{1}{3}\tan\theta$