Class XI: Physics

Chapter 8: Gravitation

Chapter Notes

Key Learnings:

1. Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F = G\frac{m_1 m_2}{r^2}$$

Where G, the universal gravitational constant, has the value 6.672 x 10^{-11} $Nm^2\ kg^{-2}$.

2. From the principle of superposition each force acts independently and uninfluenced by the other bodies. The resultant force F_R is then found by vector addition.

$$F_R = F_1 + F_2 + \ldots + F_n = \sum_{i=1}^{n} F_i$$

2. Acceleration due to gravity: $g = \frac{GM}{r^2}$

3. For small heights h above the earth's surface the value of g decreases by a factor (1-2h/R).

4. The gravitational potential energy of two masses separated by a distance r is inversely proportional to r.

5. The potential energy is never positive; it is zero only when the two bodies are infinitely far apart.

6. The gravitational potential energy associated with two particles separated by a distance r is given by

$$V = -\frac{Gm_1 m_2}{r}$$

Where V is taken to be zero at $r \to \infty$.

7. The total mechanical energy is the sum of the kinetic and potential energies. The total energy is a constant of motion.

8. If an isolated system consists of a particle of mass m mobbing with a speed v in the vicinity of a massive body of mass M, the total mechanical energy of the particle is given by

$$E = \frac{1}{2} m\, v^2 - \frac{GMm}{r}$$

If m moves in circular orbit of radius a about M, where M >> m, the total energy of the system is

$$E = -\frac{GMm}{2a}$$

9. The escape speed from the surface of the Earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$$

And has a value of 11.2 km s^{-1}

10. Kepler's law of planetary motion:

i. The orbit of the planet is elliptical with sun at one of the focus - LAW OF ORBITS.

ii. The line joining the planet to the sun sweeps out equal area in equal interval of time - LAW OF AREAS.

iii. The square of the planet's time period of revolution T, is proportional to the cube of semi major axis a.

11. A geostationary satellite moves in a circular orbit in the equatorial plane at an approximate distance of 36,000 km.

Top Formulae:

1. Newton's Law of Gravitation

$$F = \frac{Gm_1m_2}{r^2}, \qquad G = 6.67 \times 10^{-11}\ Nm^2/kg^2$$

2. Acceleration dye to Gravity

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi GR\rho$$

3. Variation of g

(a) Altitude (height) effect $g' = g\left(1 + \frac{h}{R}\right)^{-2}$

If h << R then $g' = g\left(1 - \frac{2h}{R}\right)$

(b) Effect of depth $g'' = \left(1 - \frac{d}{R}\right)$

(c) Latitude effect

4. Intensity of Gravitational Field

$$\vec{E}_g = \frac{GM}{r^2}(-\vec{r})$$

For earth $E_g = g = 9.86\ m/s^2$

5. Gravitational Potential

$$v_g = -\int_{\infty}^{r} \vec{E}_g . \vec{d}r$$

For points on out side (r >R)

$$v_g = -\frac{GM}{r}$$

For points inside it, r < R

$$v_g = -GM\left[\frac{3R^2 - r^2}{2R^3}\right]$$

6. Change in Potential Energy (P. E.) on going height h above the surface

$$\Delta U_g = mgh \qquad \text{if } h << R_e$$

In general $\Delta U_g = \dfrac{mgh}{\left(1+\dfrac{h}{R}\right)}$

7. Orbital Velocity of a Satellite

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$v_0 = \sqrt{\frac{GM}{R+h}} \quad r = h + R$$

If h << R $\quad v_0 = \sqrt{\dfrac{GM}{R}} = \sqrt{gR} = 8\ Km/sec.$

8. Velocity of Projection

Loss of K. E. = gain in P. E.

$$\frac{1}{2}mv_p^2 = -\frac{gMm}{(R+h)} - \left(-\frac{GMm}{R}\right)$$

$$v_P = \left[\frac{2GMh}{R(R+h)}\right]^{1/2} = \left[\frac{2gh}{1+\dfrac{h}{R}}\right]^{1/2} \qquad (\because GM = gR^2)$$

9. Period of Revolution

$$T = \frac{2\pi r}{v_0} = \frac{2\pi(R+h)^{3/2}}{R\sqrt{g}}$$

Or $\quad T^2 = \dfrac{4\pi^2 r^3}{GM}$

If h << R $\quad T = \dfrac{2\pi R^{3/2}}{R\sqrt{g}} = 1\dfrac{1}{2}$ hr.

10. Kinetic Energy of Satellite

$$K.E. = \frac{GMm}{2r} = \frac{1}{2mv_0^2}$$

11. P.E. of Satellite

$$U = -\frac{GMm}{r}$$

12. Binding energy of Satellite = $\dfrac{1}{2}\dfrac{GMm}{r}$

13. Escape Velocity

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = R\sqrt{\frac{8\pi Gd}{3}}$$

$$v_e = v_0\sqrt{2}$$

14. Effective Weight in a Satellite

$$w = 0$$

Satellite behaves like a free fall body

15. Kepler's Laws for Planetary Motion

(a) Elliptical orbit with sun at one focus

(b) Areal velocity constant dA/ dt = constant

(c) $T^2 \propto r^3$. $r = (r_1 + r_2)/ 2$