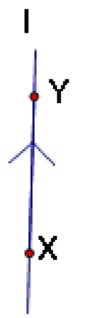
Class XII: Mathematics
Chapter 9: Vector Algebra
Chapter Notes

Key Concepts

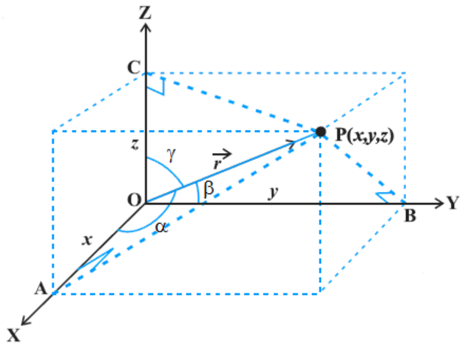
1. A quantity that has magnitude as well as direction is called a vector.

2. A directed line segment is called a vector.



The point X from where the vector starts is called the initial point and the point Y where it ends is called the terminal point.

3. For vector $\overrightarrow{XY}$, magnitude =distance between X and Y and is denoted by $|\overrightarrow{XY}|$, which is greater then or equal to zero.
4. The distance between the initial point and the terminal point is called the magnitude of the vector.
5. The position vector of point P $\equiv (x_1,y_1,z_1)$ with respect to the origin is given by: $\overrightarrow{OP} = \vec{r} = \sqrt{x^2 + y^2 + z^2}$
6. If the position vector $\overrightarrow{OP}$ of a point P makes angles α, β and γ with x, y and z axis respectively, then α, β and γ are called the **direction angles** and $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the **Direction cosines** of the position vector $\overrightarrow{OP}$.
7. Then $\lambda = \cos \alpha$, $m=\cos \beta$, $n=\cos \gamma$ are called the direction cosines of $\vec{r}$.



8. The numbers lr,mr, nr, proportional to l,m,n are called direction ratios of vector $\vec{r}$, and are denoted by a,b,c.
In general, $\lambda^2+m^2+n^2=1$ but $a^2+b^2+c^2 \neq 1$

9.Vectors can be classified on the basis of position and magnitude. On the basis of magnitude vectors are: zero vector and unit vector. On the basis of position, vectors are: coinitial vectors, parallel vectors,free vectors, and collinear vectors .

10. Zero vector is a vector whose initial and terminal points coincide and is denoted by $\vec{0}$. $\vec{0}$ is called the additive identity.

11. The Unit vector has a magnitude equal to 1.A unit vector in the direction of the given vector $\vec{a}$ is denoted by $\hat{a}$.

12. Co initial vectors are vectors having the same initial point.

13. Collinear vectors are parallel to the same line irrespective of their magnitudes and directions.

14. Two vectors are said to be parallel if they are non zero scalar multiples of one another.

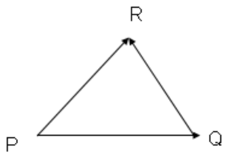
15. Equal vectors as the name suggests, are vectors which have same magnitude and direction irrespective of their initial points.

16. The negative vector of a given vector $\vec{a}$ is a vector which has the same magnitude as $\vec{a}$ but the direction is opposite of $\vec{a}$

17. A vector whose initial position is not fixed is called free vector.

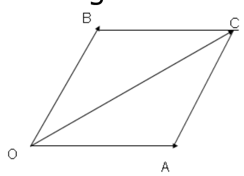
18. Two vectors can be added using the triangle law and parallelogram law of vector addition Vector addition is both commutative as well as associative

19. **Triangle Law of Vector Addition**: Suppose two vectors are represented by two sides of a triangle in sequence, then the third closing side of the triangle represents the sum of the two vectors



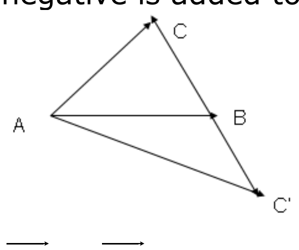
$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

20. **Parallelogram Law of Vector Addition**: If two vectors $\vec{a}$ and $\vec{b}$ are represented by two adjacent sides of a parallelogram in magnitude and direction, then their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram.



$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$

21. Difference of vectors: To subtract a vector $\overrightarrow{BC}$ from vector $\overrightarrow{AB}$ its negative is added to $\overrightarrow{AB}$



$\overrightarrow{BC'} = -\overrightarrow{BC}$

$\overrightarrow{AB} + \overrightarrow{BC'} = \overrightarrow{AC'}$

$\Rightarrow \overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AC'}$

22. If $\vec{a}$ is any vector and k is any scalar then scalar product of $\vec{a}$ and k is $k\vec{a}$.

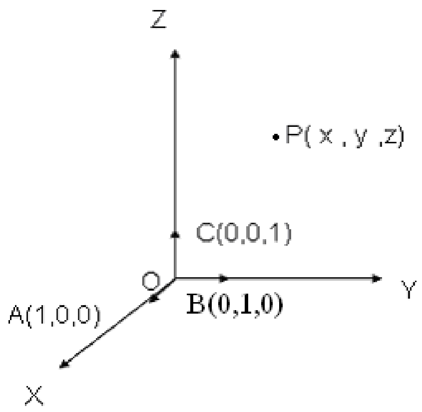
$k\vec{a}$ is also a vector , collinear to the vector $\vec{a}$.

$k>0 \Rightarrow k\vec{a}$ has the same direction as $\vec{a}$.

$k<0 \Rightarrow k\vec{a}$ has opposite direction as $\vec{a}$.

Magnitude of $k\vec{a}$ is |k| times the magnitude of vector $k\vec{a}$.

23. Unit vectors along OX, OY and OZ are denoted by of $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.



Vector $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is called the component form of vector r
Here, x, y and z are called the scalar components of $\vec{r}$ in the directions of $\hat{i}, \hat{j}$ and $\hat{k}$, and $x\hat{i}, y\hat{j}$ are $z\hat{k}$ are called the vector components of vector r along the respective axes.

24Two vectors are $\vec{a}$ and $\vec{b}$ are collinear $\Leftrightarrow$ $\vec{b} = k\,\vec{a}$, where k is a non zero scalar. Vectors $\vec{a}$ and $k\vec{a}$ are always collinear.

25. If $\vec{a}$ and $\vec{b}$ are equal then $|\vec{a}| = |\vec{b}|$.

26. If P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) are any two points then vector joining P and Q is, $\overrightarrow{PQ}$=position vector of Q – position vector of P i.e $\overrightarrow{PQ}=(x_2 - x_1)\hat{i}+(y_2 - y_1)\hat{j}+(z_2 - z_1)\hat{k}$

27.

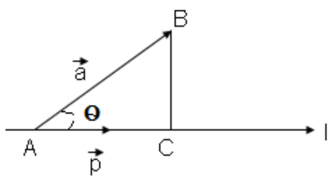
The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio m : n

· (i) internally, is given by $\dfrac{n\vec{a}+m\vec{b}}{m+n}$

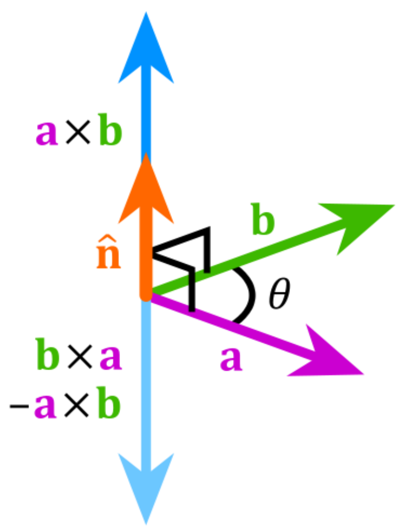
(ii) externally, is given by $\dfrac{m\vec{b}-n\vec{a}}{m-n}$

28. Scalar or dot product of two non zero vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\cdot\vec{b}$ given by $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between vectors $\vec{a}$ and $\vec{b}$ and $0 \le \theta \le \pi$

29.Projection of vector **AB,** making an angle of θ with the line L, on line L is vector $\vec{P} = |\overrightarrow{AB}|$ **cos** θ



30. The vector product of two non zero vectors $\vec{a}$ and $\vec{b}$ denoted by $\vec{a}\times\vec{b}$ is defined as $\vec{a}\times\vec{b} = |a||b|\sin\theta\hat{n}$ where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$,
Here $\vec{a}, \vec{b}$ and $\hat{n}$ form a right handed system.



31. Area of a parallelogram is equal to modulus of the cross product of the vectors representing its adjacent sides.
32. Vector sum of the sides of a triangle taken in order is zero.

Key Formulae

1. Properties of addition of vectors

1) vector addition is commutative

$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

2) vector addition is associative.

$$\vec{a}+(\overrightarrow{b+c})=(\overrightarrow{a+b})+\vec{c}$$

3) $\vec{0}$ is additive identity for vector addition

$$\vec{a}+\vec{0}=\vec{0}+\vec{a}=\vec{a}$$

2. Magnitude or Length of vector $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ is $|\vec{r}|=\sqrt{x^2+y^2+z^2}$

3. Vector addition in Component Form: Given $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ then $\vec{r}_1+\vec{r}_2=(x_1+x_2)\hat{i}+(y_1+y_2)\hat{j}+(z_1+z_2)\hat{k}$

4. Difference of vectors: Given $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ then $\vec{r}_1-\vec{r}_2=(x_1-x_2)\hat{i}+(y_1-y_2)\hat{j}+(z_1-z_2)\hat{k}$

5. Equal Vectors: Given $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ then $\vec{r}_1=\vec{r}_2$ if and only if $x_1=x_2; y_1=y_2; z_1=z_2$

6. Multiplication of $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ with scalar k is given by

$k\vec{r}=(kx)\hat{i}+(ky)\hat{j}+(kz)\hat{k}$

7. For any vector $\vec{r}$ in component form $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then x, y, z are the direction ratios of $\vec{r}$ and $\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}$ are its direction cosines.

8. Let $\vec{a}$ and $\vec{b}$ be any two vectors and k and m being two scalars then

(i) $k\vec{a}+m\vec{a}=(k+m)\vec{a}$

(ii) $k(m\vec{a})=(km)\vec{a}$

(iii) $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$

9. Vectors $\vec{r}_1=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{r}_2=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ are collinear if

$$\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)=k\left(x_2\hat{i}+y_2\hat{j}+z_2\hat{k}\right)$$

i.e $x_1=kx_2$; $y_1=ky_2$; $z_1=k z_2$

or $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = k$

10. Scalar product of vectors a and b is $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between vectors

11.Properties of Scalar product

(i) $\vec{a}\cdot\vec{b}$ is a real number.

(ii)If $\vec{a}$ and $\vec{b}$ are non zero vectors then $\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$.

(iii) Scalar product is commutative : $\vec{a}\cdot\vec{b} = \vec{b}.\vec{a}$

(iv)If $\theta = 0$ then $\vec{a}\cdot\vec{b} = |\vec{a}|.|\vec{b}|$

(v) If $\theta = \pi$ then $\vec{a}\cdot\vec{b} = -|\vec{a}|.|\vec{b}|$

(vi) scalar product distribute over addition

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors, then

$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

(vii)Let $\vec{a}$ and $\vec{b}$ be two vectors, and λ be any scalar.

Then $(\lambda\vec{a}).\vec{b} = (\lambda\vec{a}).\vec{b} = \lambda(\vec{a}.\vec{b}) = a.(\lambda\vec{b})$

12. Angle between two non zero vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

or $\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}\right)$.

13. For unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ and $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

14. Unit vector in the direction of vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is $\frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$

15

Projection of a vector $\vec{a}$ on other vector $\vec{b}$ is given by:

$\vec{a}.\hat{b} = \vec{a}.\left(\frac{\vec{b}}{|\vec{b}|}\right) = \frac{1}{|\vec{b}|}\left(\vec{a}.\vec{b}\right)$

16. Cauchy-Schwartz Inequality:

$|\vec{a}.\vec{b}| \le |\vec{a}|.|\vec{b}|$

17.Triangle Inequality: $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$

18. Vector r product of vectors a and b is $\vec{a}\times\vec{b}=|a||b|\sin\theta\hat{n}$.

19. Properties of Vector Product:

(i) $\vec{a}\times\vec{b}$ is a vector

(ii) If $\vec{a}$ and $\vec{b}$ are non zero vectors then $\vec{a}\times\vec{b}=0$ iff $\vec{a}$ and $\vec{b}$ are collinear i.e

$\vec{a}\times\vec{b}=0 \Leftrightarrow \vec{a}||\vec{b}$ Either θ =0 or $\theta = \pi$

(iii) If $\theta = \dfrac{\pi}{2}$, then $|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|$

(iv) vector product distribute over addition

If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors and λ is a scalar, then

(i) $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$

(ii) $\lambda(\vec{a}\times\vec{b})=(\lambda\vec{a})\times\vec{b}=\vec{a}\times(\lambda\vec{b})$

(v) If we have two vectors $\vec{a}$ and $\vec{b}$ given in component form as

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

then $\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(vi) For unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$

$\hat{i}\times\hat{i} = \hat{j}\times.\hat{j} = \hat{k}\times.\hat{k} = 0$ and $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}.\hat{k} = \hat{i}$, $\hat{k}.\hat{i} = \hat{j}$

(vii) $\vec{a}\times\vec{a}=\vec{0}$ as $\theta=0 \Rightarrow \sin\theta=0$

$\vec{a}\times(-\vec{a})=\vec{0}$ as $\theta=\pi \Rightarrow \sin\theta=0$

$\vec{a}\perp\vec{b} \Rightarrow \theta = \dfrac{\pi}{2} \Rightarrow \sin\theta=1 \Rightarrow \vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\;\hat{n}$

(vii) . Angle between two non zero vectors $\vec{a}$ and $\vec{b}$ is given by

$\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$ or $\theta = \sin^{-1}\left(\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right)$