Class XII: Math
Chapter 12: Linear Programming

Chapter Notes

Key Concepts

1. Linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtainable under those conditions.
2. Linear programming is part of a very important area of mathematics called "optimisation techniques.
3. Type of problems which seek to maximise (or minimise) profit (or cost) form a general class of problems called optimisation problems.

4. A problem which seeks to maximise or minimise a linear function subject to certain constraints as determined by a set of linear inequalities is called an optimisation problem.

5. A linear programming problem may be defined as the problem of maximising or minimising a linear function subject to linear constraints. The constraints may be equalities or inequalities.

6. **Objective Function:** Linear function $Z = ax + by$, where a, b are constants, x and y are decision variables, which has to be maximised or minimised is called a linear objective function. Objective function represents cost, profit, or some other quantity to be maximised of minimised subject to the constraints.

7. The linear inequalities or equations that are derived from the application problem are problem constraints.

8. The linear inequalities or equations or restrictions on the variables of a linear programming problem are called constraints.

9. The conditions $x \geq 0$, $y \geq 0$ are called non – negative restrictions. Non - negative constraints included because x and y are usually the number of items produced and one cannot produce a negative number of items. The smallest number of items one could produce is zero. These are not (usually) stated, they are implied.

10. A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear function (called objective function) of several variables (say x and y), subject to the conditions that the variables are non – negative and satisfy a set of linear inequalities (called linear constraints).

11. In **Linear Programming** the term **linear** implies that all the mathematical relations used in the problem are linear and **Programming** refers to the method of determining a particular programme or plan of action.

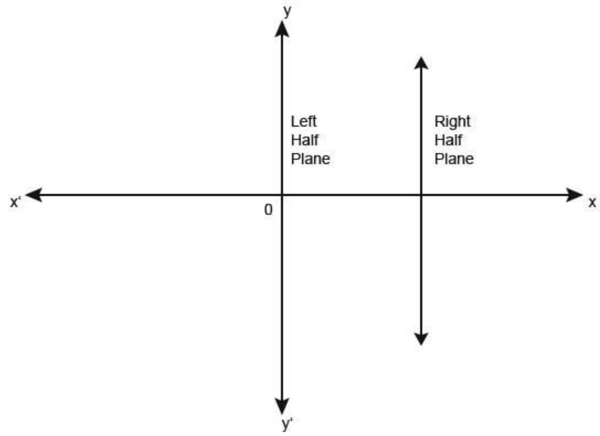
12. Forming a set of linear inequalities (constrains) for a given situation is called formulation of the linear programming problem or LPP.

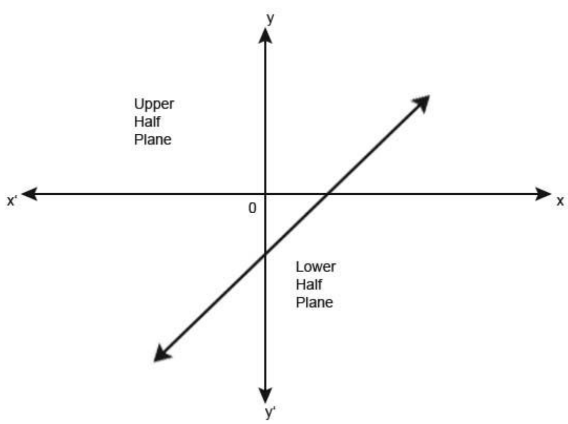
13. **Mathematical Formulation of Linear Programming Problems**

Step I: In every LPP certain decisions are to be made. These decisions are represented by decision variables. These decision variables are those quantities whose values are to be determined. Identify the variables and denote them by x_1, x_2, x_{3}, ... Or x,y and z etc
Step II: Identify the objective function and express it as a linear function of the variables introduced in step I.
Step III: In a LPP, the objective function may be in the form of maximising profits or minimising costs. So identify the type of optimisation i.e., maximisation or minimisation.
Step IV: Identify the set of constraints, stated in terms of decision variables and express them as linear inequations or equations as the case may be.
Step V: Add the non-negativity restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation.

14. General LPP is of the form
Max (or min) $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ $c_1, c_2, \ldots . c_n$ are constants
$x_1, x_2, \ldots . x_n$ are called decision variable.
s.t $Ax \leq (\geq) B$ and $x_i \geq 0$

15. A linear inequality in two variables represents a half plane geometrically.
    Types of half planes





16. The common region determined by all the constraints including non – negative constraints x, y ≥ 0 of a linear programming problem is called the **feasible region** (or solution region) for the problem. The region other than feasible region is called an infeasible region.
17. Points within and on the boundary of the feasible region represent **feasible solution** of the constraints.
18. Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
19. Any point outside the feasible region is called an infeasible solution.

20. A corner point of a feasible region is the intersection of two boundary lines.
21. A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle.
22. **Corner Point Theorem 1:** Let R be the feasible region (convex polygon) for a linear programming problem and let Z = ax + by be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.
23. **Corner Point Theorem 2:** Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objective function. If R is bounded, then the objectives function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R.
24. If R is **unbounded**, then a maximum or a minimum value of the objectives functions may not exist.
25. The **graphical method** for solving linear programming problems in two unknowns is as follows.

    A. Graph the feasible region.

    B. Compute the coordinates of the corner points.

    C. Substitute the coordinates of the corner points into the objective function to see which gives the optimal value.

    D. When the feasible region is bounded, M and m are the maximum and minimum values of Z.

    E. If the feasible region is not bounded, this method can be misleading: optimal solutions always exist when the feasible region is bounded, but may or may not exist when the feasible region is unbounded.

    (i) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

    (ii) Similarly, m is the minimum value of Z,if the open half plane determined by ax+by<m has no point in common with the feasible region. Otherwise, Z has no minimum value.

26. Points within and on the boundary of the feasible region represent feasible solutions of the constraints.

27. If two corner points of the feasible region are both optimal solutions of the same type, i.e., both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

28. **Types of Linear Programming Problems**

1 **Manufacturing problems** : Problems dealing in finding the number of units of different products to be produced and sold by a firm when each product requires a fixed manpower, machine hours, labour hour per unit of product in order to make maximum profit.

2 **Diet problem:** Problems, dealing in finding the amount of different kinds of nutrients which should be included in a diet so as to minimise the cost of the desired diet such that it contains a certain minimum amount of each constituent/nutrients.

3 **Transportation problems:** Problems dealing in finding the transportation schedule of the cheapest way to transpor a product from plants/factories situated at different locations to different markets.

29. Advantages of LPP

(i) Linear programming technique helps to make the best possible use of available productive resources (such as time, labour, machines etc.)

(ii) A significant advantage of linear programming is highlighting of such bottle necks.

30. Disadvantages of LPP

(i) Linear programming is applicable only to problems where the constraints and objective functions are linear i.e., where they can be expressed as equations which represent straight lines.

(ii) Factors such as uncertainty, weather conditions etc. are not taken into consideration.