Class XII: Mathematics
Chapter : Probability
Chapter Notes

Key Concepts

1. The probability that event B will occur if given the knowledge that event A has already occurred is called conditional probability. It is denoted as P(B|A).

2. Conditional probability of B given A has occurred P(B|A) is given by the ratio of number of events favourable to both A and B to number of events favourable to A.

3. E and F be events of a sample space S of an experiment, then
   (i). $P(S|F) = P(F|F)=1$
   (ii) For any two events A and B of sample space S if F is another event such that $P(F) \neq 0$
   $P((A \cup B)|F) = P(A|F)+P(B|F)-P((A \cap B)|F)$
   (iii) $P(E'|F) = 1-P(E|F)$

4. Two events A and B are independent if and only if the occurrence of A does not depend on the occurrence of B and vice versa.

5. If events A and B are independent then $P(B|A) = P(A)$ and $P(A|B)=P(A)$

6. Three events A,B, C are independent if they are pair wise independent i.e
   $P(A \cap B) = P(A).P(B)$, $P(A \cap C) = P(A). P(C)$, $P(B \cap C) = P(B).P(C)$

7. Three events A,B, C are independent if
   $P(A \cap B \cap C) = P(A). P(B). P(C)$
Independence implies pair wise independence, but not conversely.

8. In the case of independent events, event A has no effect on the probability of event B so the conditional probability of event B given event A has already occurred is simply the probability of event B, $P(B|A)=P(B)$.

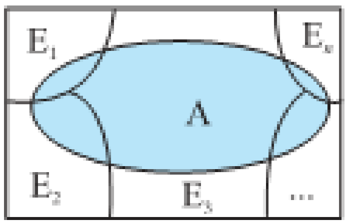
9. If E and F are independent events then so are the events
   (i)E' and F
   (ii)E and F'
   (iii)E' and F'

10. If A and B are events such that $B \neq \varphi$ then B is said to affect A
i) favourably if $P(A|B) > P(A)$

ii) unfavourably if P(A|B) < P(A)
iii) not at all if P(A|B) = P(A).

11. Two events E and F are said to be dependent if they are not independent, i.e. if $P(E \cap F) \neq P(E).P(F)$

12. The events $E_1, E_2, ..., En$ represent a partition of the sample space S if



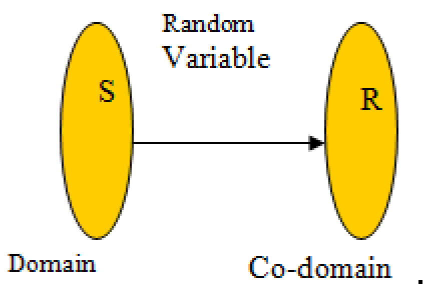
(1) They are pair wise disjoint,
(2) They are exhaustive and
(3) They have nonzero probabilities.

13.The events $E_1, E_2,...,E_n$ are called hypotheses. The probability $P(E_i)$ is called the priori probability of the hypothesis E_i. The conditional probability $P(E_i| A)$ is called a posteriori probability of the hypothesis E_i

14. Bayes' Theorem is also known as the formula for the probability of **"causes".**

15. When the value of a variable is the outcome of a random experiment, that variable is a **random variable.**

16. A random variable is a function that associates a real number to each element in the sample space of random experiment.



17. A random variable which can assume only a finite number of values or countably infinite values is called a discrete random variable.
In experiment of tossing three coins a random variable X representing number of heads can take values 0, 1, 2, 3.

18. A random variable which can assume all possible values between certain limits is called a continuous random variable.
Examples include height, weight etc.

19. The probability distribution of a random variable X is the system of numbers

$$\begin{array}{lcllll} X & : & x_1 & x_2 & \ldots.. & x_n \\ P(X) & : & p_1 & p_2 & \ldots.. & p_n \end{array}$$

where $p_i > 0, \sum_{i=1}^{n} p_i = 1, i = 1, 2, 3, \ldots., n$

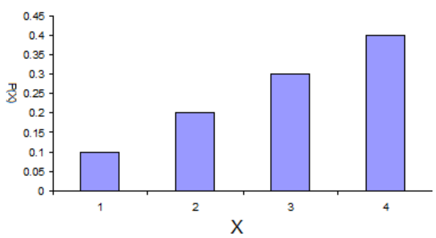
The real numbers $x_1, x_2, \ldots, x_n$ are the possible values of the random variable X and p_i (i = 1,2,..., n) is the probability of the random variable X taking the value x_i i.e. $P(X = x_i) = p_i$

20. In the probability distribution of x each probability pi is non negative, and sum of all probabilities is equal to 1.

21. Probability distribution of a random variable x can be represented using bar charts.

X	**1**	**2**	**3**	**4**
P(X)	**.1**	**.2**	**.3**	**.4**

Tabular Representation



Graphical Representation

22. The expected value of a random variable indicates its average or central value.

23. The expected value of a random variable indicates its average or central value. It is a useful summary value of the variable's distribution.

24. Let X be a discrete random variable which takes values $x_1, x_2, x_3, \ldots x_n$ with probability $p_i = P\{X = x_i\}$, respectively. The mean of X, denoted by μ, is summation pixi

25. Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes: success or failure.

(iv) The probability of success remains the same in each trial.

26. **Binomial distribution** is the discrete probability distribution of the number of successes in a sequence of *n* independent binomial experiments, each of which yields success with probability p.

27. **Bernoulli experiment** is a random experiment whose trials have two outcomes that are mutually exclusive: they are, termed, success and failure.

28. Binomial distribution with n-Bernoulli trials, with the probability of success in each trial as p, is denoted by B (n, p). n and p are called the parameters of the distribution.

29. The random variable *X* follows the binomial distribution with parameters *n* and *p*, we write *XK* ~ B(*n*, *p*). The probability of getting exactly *k* successes in *n* trials is given by the probability mass function
$P(X = k) = {}^{n}C_{k}\, q^{n-k}\, p^{k}$

30. Equal means of two probability distributions does not imply same distributions.

Key Formulae

1. $0 \le P(B|A) \le 1$

2. If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$$P(E\,|\,F) = \frac{n(E \cap F)}{n(F)} \text{ provided } P(F) \neq 0 \text{ or}$$

$$P(F\,|\,E) = \frac{n(E \cap F)}{n(E)} \text{ provided } P(E) \neq 0$$

3. **Multiplication Theorem:**

(a) For two events

Let E and F be two events associated with a sample space S.

$P(E \cap F) = P(E)\,P(F|E) = P(F)\,P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.

(b) For three events:
If E, F and G are three events of sample space S,

$P(E \cap F \cap G) = P(E)\,P(F|E)\,P(G|(E \cap F)) = P(E)\,P(F|E)\,P(G|EF)$

4. **Multiplication theorem for independent Events**
   (i) $P(E \cap F) = P(E)P(F)$
   (ii) $P(E \cap F \cap G) = P(E)P(F)P(G)$

5. Let E and F be two events associated with the same random experiment Two events E and F are said to be independent, if
   (i) $P(F|E) = P(F)$ provided $P(E) \neq 0$ and
   (ii) $P(E|F) = P(E)$ provided $P(F) \neq 0$
   (iii) $P(E \cap F) = P(E) \cdot P(F)$

6. Occurrence of atleast one of the two events A or B
   $P(A \cup B) = 1 - P(A')P(B')$

7. A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S if
(a) $E_i \cap E_j = \phi$, $i \neq j$, $i, j = 1, 2, 3, \ldots, n$
(b) $E_1 \cup E_2 \cup E_n = S$
(c) $P(E_i) > 0$ for all $i = 1, 2, \ldots, n$.

8. **Theorem of Total Probability**
Let $\{E_1, E_2, \ldots, E_n\}$ be a partition of the sample space S, and suppose that each of the events $E_1, E_2, \ldots, E_n$ has nonzero probability of occurrence. Let A be any event associated
with S, then
$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + \ldots + P(E_n)\,P(A|E_n)$
$$= \sum_{j=1}^{n} P(E_j)P(A \mid E_j)$$

9. **Bayes' Theorem**
If $E_1, E_2, \ldots, E_n$ are n non-empty events which constitute a partition of sample space S and
A is any event of nonzero probability, then
$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A \mid E_j)}$$ for any I =1,2,3,…n

10. The mean or expected value of a random variable X, denoted by E(X) or μ is defined as
$$E(X) = \mu = \sum_{i=1}^{n} x_i p_i$$

11. The variance of the random variable X, denoted by Var (X) or σ_x^2 is defined as

$$\sigma_x^2 = \text{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i) = E(X - \mu)^2$$

Var (X) = $E(X^2) - [E(X)]^2$

12. Standard Deviation of random variable X:

$$\sigma_x = \sqrt{\text{Var}(X)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)}$$

13. For Binomial distribution B (n, p),
P (X = x) = $^nC_x\, q^{n-x}\, p^x$, x = 0, 1,..., n (q = 1 – p)

14. Mean and Variance of a variable X following Binomial distribution
E (X) = μ = np
Var (X) = npq
Where n is number of trials, p = probability of success
q = probability of failures
15. Standard Deviation of a variable X following Binomial distribution

$$\sigma_x = \sqrt{\text{Var}(X)} = \sqrt{npq}$$