Class XII

Mathematics

Chapter:1

Relations and Functions
Points to Remember

Key Concepts

1. A relation R between two non empty sets A and B is a subset of their Cartesian Product $A \times B$. If A = B then relation R on A is a subset of $A \times A$

2. If (a, b) belongs to R, then a is related to b, and written as a R b If (a, b) does not belongs to R then a $\not R$ b.

3. Let R be a relation from A to B.
Then Domain of $R \subset A$ and Range of $R \subset B$ co domain is either set B or any of its superset or subset containing range of R

4. A relation R in a set A is called **empty** relation, if no element of A is related to any element of A, i.e., $R = \phi \subset A \times A$.

5. A relation R in a set A is called **universal** relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

6. A relation R in a set A is called

   a. **Reflexive**, if $(a, a) \in R$, for every $a \in A$,

   b. **Symmetric**, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

   c. **Transitive**, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, or all $a_1, a_2, a_3 \in A$.

7. A relation R in a set A is said to be an **equivalence relation** if R is reflexive, symmetric and transitive.

8. The empty relation R on a non-empty set *X* (i.e. a R b is never true) is not an equivalence relation, because although it is vacuously symmetric and transitive, it is not reflexive (except when *X* is also empty)

9. Given an arbitrary equivalence relation R in a set X, R divides X into mutually disjoint subsets S_i called partitions or subdivisions of X satisfying:

- All elements of S_i are related to each other, for all i
- No element of S_i is related to S_j ,if i $\neq$ j
- $\bigcup_{i=1}^{n} S_j$ =X and $S_i \cap S_j = \phi$, if i $\neq$ j
- The subsets S_j are called Equivalence classes.

10. A function from a non empty set A to another non empty set B is a correspondence or a rule which associates every element of A to a unique element of B written as
f:A $\rightarrow$ B s.t f(x) = y for all x $\in$ A, y $\in$ B. All functions are relations but converse is not true.

11. If f: A $\rightarrow$ B is a function then set A is the domain, set B is co-domain and set {f(x):x $\in$ A } is the range of f. Range is a subset of codomain.

12. f: A $\rightarrow$ B is one-to-one if

    For all x, y $\in$ A f(x) = f(y) $\Rightarrow$ x = y or x $\neq$ y $\Rightarrow$ f(x) $\neq$ f(y)

    A one- one function is known as injection or an Injective Function.

    Otherwise, f is called many-one.

13. f: A $\rightarrow$ B is an onto function ,if for each b $\in$ B there is atleast one a $\in$ A such that f(a) = b

    i.e if every element in B is the image of some element in A, f is onto.

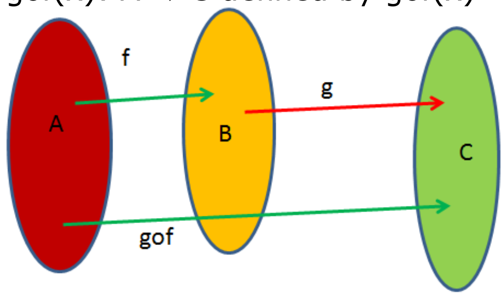
14. A function which is both one-one and onto is called a bijective function or a bijection.

15. For an onto function range = co-domain.

16. A one – one function defined from a finite set to itself is always onto but if the set is infinite then it is not the case.

17. Let f : $A \rightarrow B$ and g : $B \rightarrow C$ be two functions. Then the composition of f and g, denoted by gof is defined as the function gof: A $\rightarrow$ C given by

    gof(x): A $\rightarrow$ C defined by gof(x) = g(f(x)) $\forall$ x $\in$ A



Composition of f and g is written as gof and not fog
gof is defined if the range of f $\subseteq$ domain of f and fog is defined if range of g $\subseteq$ domain of f

18. Composition of functions is not commutative in general

    fog(x) ≠ gof(x).Composition is associative

    If f: X$\rightarrow$Y, g: Y$\rightarrow$Z and h: Z$\rightarrow$S are functions then

    ho(g o f)=(h o g)of

19. A function f: X $\rightarrow$ Y is defined to be invertible, if there exists a function

    g : Y $\rightarrow$ X such that gof = I_X and fog = I_Y. The function g is called the inverse of f and is denoted by f^{-1}

20. If f is invertible, then f must be one-one and onto and conversely, if f is one- one and onto, then f must be invertible.

21. If f:A $\rightarrow$ B and g: B $\rightarrow$C are one-one and onto then gof: A $\rightarrow$ C is also one-one and onto. But If g o f is one –one then only f is one –one g may or may not be one-one. If g o f is onto then g is onto f may or may not be onto.

22. Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be two invertible functions. Then gof is also

    Invertible with $(gof)^{-1} = f^{-1} o\ g^{-1}$.

23. If f: R$\rightarrow$ R is invertible,

    f(x)=y, then $f^{-1}(y)$=x and $(f^{-1})^{-1}$ is the function f itself.

24. A binary operation * on a set A is a function from A X A to A.

25. Addition, subtraction and multiplication are binary operations on R, the set of real numbers. Division is not binary on R, however, division is a binary operation on R-{0}, the set of non-zero real numbers

26. A binary operation $*$ on the set X is called commutative, if $a*b= b*a$, for every a,b$\in$X

27. A binary operation $*$ on the set X is called associative, if $a*(b*c) =(a*b)*c$, for every a, b, c$\in$X

28. An element $e \in A$ is called an **identity** of *A* with respect to *, if for each $a \in A$, $a * e = a = e * a$.
    The identity element of (A, *) if it exists, is **unique**.

29.Given a binary operation $*$ from $A\times A \rightarrow A$, with the identity element e in A, an element $a \in A$ is said to be invertible with respect to the operation $*$, if there exists an element b in A such that $a*b=e=b*a$, then b is called the inverse of a and is denoted by a^{-1}.

30.If the operation table is symmetric about the diagonal line then, the operation is commutative.

*	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

The operation * is commutative.

31. Addition '+' and multiplication '·' on N, the set of natural numbers are binary operations But subtraction '–' and division are not since (4, 5) = 4 - 5 = -1 $\notin N$ and 4/5 =.8 $\notin N$