Class XII

Mathematics

Chapter:2

Inverse Trigonometric Functions

Points to Remember

Key Concepts

1. Inverse trigonometric functions map real numbers back to angles.
2. Inverse of sine function denoted by $\sin^{-1}$ or arc sin(x) is defined on [-1,1] and range could be any of the intervals
$\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.
3. The branch of $\sin^{-1}$ function with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is the principal branch.

   So $\sin^{-1}$: $[-1,1] \rightarrow \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
4. The graph of $\sin^{-1} x$ is obtained from the graph of sine x by interchanging the x and y axes
5. Graph of the inverse function is the mirror image (i.e reflection) of the original function along the line y = x.
6. Inverse of cosine function denoted by $\cos^{-1}$ or arc cos(x) is defined in [-1,1] and range could be any of the intervals $[-\pi,0]$, $[0,\pi]$, $[\pi,2\pi]$.
So, $\cos^{-1}$: $[-1,1] \rightarrow [0,\pi]$.

7. The branch of $\tan^{-1}$ function with range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is the principal branch. So $\tan^{-1}$: $R \rightarrow \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

8. The principal branch of $\operatorname{cosec}^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$-{0}.

   $\operatorname{cosec}^{-1} x$:R-(-1,1) $\rightarrow \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$-{0}.

9. The principal branch of $\sec^{-1} x$ is $[0,\pi]$-{ $\frac{\pi}{2}$ }.

   $\sec^{-1} x$:R-(-1,1) $\rightarrow[0,\pi]$-{ $\frac{\pi}{2}$ }.

10. $\cot^{-1}$ is defined as a function with domain R and range as any of the intervals $(-\pi,0)$, $(0,\pi)$,$(\pi,2\pi)$. The principal branch is $(0,\pi)$

    So $\cot^{-1}$: $R \rightarrow (0,\pi)$

11. The value of an inverse trigonometric function which lies in the range of principal branch is called the principal value of the inverse trigonometric functions.

12. Inverse of a function is not equal to the reciprocal of the function.

13. Properties of inverse trigonometric functions are valid only on the principal value branches of corresponding inverse functions or wherever the functions are defined.

Key Formulae

1.Domain and range of Various inverse trigonometric Functions

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \text{cosec}^{-1} x$	$R - (-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$

2. **Self Adjusting property**

$\sin(\sin^{-1}x)=x$; $\sin^{-1}(\sin x) = x$

$\cos(\cos^{-1} x)=x; \cos^{-1}(\cos x)=x$

$\tan(\tan^{-1} x)=x; \tan^{-1}(\tan x)=x$

Holds for all other five trigonometric ratios as well.

3. **Reciprocal Relations**

$\sin^{-1}\left(\frac{1}{x}\right)=\text{cosec}^{-1} x$, $x \geq 1$ or $x \leq -1$

$\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x, x \geq 1$ or $x \leq -1$

$\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x, x > 0$

4. Even and Odd Functions

(i) $\sin^{-1}(-x) = -\sin^{-1}(x),\ x \in [-1,1]$

(ii) $\tan^{-1}(-x) = -\tan^{-1}(x),\ x \in R$

(iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x,\ |x| \geq 1$

(iv) $\cos^{-1}(-x) = \pi - \cos^{-1}x, x \in [-1,1]$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x, |x| \geq 1$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1}x, x \in R$

5. Complementary Relations

(i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, x \in [-1,1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, x \in R$

(iii) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}, |x| \geq 1$

6. Sum and Difference Formuale

(i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right), xy > -1$

(iii) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}]$

(iv) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$

(v) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}]$

(vi) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}]$

(vii) $\cot^{-1}x + \cot^{-1}y = \cot^{-1}\left(\frac{xy-1}{x+y}\right)$

(viii) $\cot^{-1}x - \cot^{-1}y = \cot^{-1}\left(\frac{xy+1}{y-x}\right)$

7. Double Angle Formuale

(i) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}, |x| \le 1$

(ii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}, x \ge 0$

(iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}, -1 < x < 1$

(iv) $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}), \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(v) $2\cos^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}), \frac{1}{\sqrt{2}} \le x \le 1$

8. Conversion Properties

(i) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$

$$= \tan^{-1}\frac{x}{x\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$$

(ii) $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$

$$= \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$$

(iii) $\tan^{-1}x = \sin^{-1} = \frac{x}{\sqrt{1-x^2}}$

$$= \cos^{-1}\frac{x}{\sqrt{1+x^2}} = \sec^2\sqrt{1+x^2}$$

$$= \operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}$$

Properties are valid only on the values of x for which the inverse functions are defined.