# Class XII: Maths

# Chapter 3: Matrices

# Chapter Notes

## Top Definitions

1. Matrix is an ordered rectangular array of numbers (real or complex) or functions or names or any type of data. The numbers or functions are called the elements or the entries of the matrix.

2. The horizontal lines of elements are said to constitute, rows of the matrix and the vertical lines of elements are said to constitute, columns of the matrix.

3. A matrix is said to be a column matrix if it has only one column.

$A = [a_{ij}]_{m \times 1}$ is a column matrix of order m x 1

4. A matrix is said to be a row matrix if it has only one row.

$B = [b_{ij}]_{1 \times n}$ is a row matrix of order 1 x n.

5. A matrix in which the number of rows is equal to the number of columns is said to be a square matrix. A matrix of order "$m \times n$" is said to be a square matrix if m = n and is known as a square matrix of order 'n'.

$A = [a_{ij}]_{m \times n}$ is a square matrix of order m.

6. If $A = [a_{ij}]$ is a square matrix of order n, then elements $a_{11}, a_{22}, \ldots, a_{nn}$ are said to constitute the diagonal, of the matrix A

7. A square matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if all its non diagonal elements are zero, that is a matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

8. A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}]_{n \times n}$ is said to be a scalar matrix if

$b_{ij} = 0$, when $i \neq j$

$b_{ij} = k$, when $i = j$, for some constant k.

9. A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an identity matrix. A square matrix

$A = [a_{ij}]_{n \times n}$ is an identity matrix, if $a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

10. A matrix is said to be zero matrix or null matrix if all its elements are zero.

11. Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) They are of the same order

(ii) Each elements of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

12. If $A = [a_{ij}]$ be an m x n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A′ or ($A^T$).

13. For any square matrix A with real number entries, A + A′ is a symmetric matrix and A – A′ is a skew symmetric matrix.

14. If $A = \left[a_{ij}\right]_{nxn}$ is an $n \times n$ matrix such that $A^T = A$, then A is called symmetric matrix. In a symmetric matrix, $a_{ij} = a_{ji}$ for all i and j

15. If $A = \left[a_{ij}\right]_{nxn}$ is an $n \times n$ matrix such that $A^T = -A$, then A is called skew

symmetric matrix. In a skew symmetric matrix, $a_{ij} = -a_{ji}$

If i=j, then $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$

16. Let A and B be two square matrices of order n such that AB=BA=I. Then A is called inverse of B and is denoted by B=$A^{-1}$. If B is the inverse of A , then A is also the inverse of B.

17. If A and B are two invertible matrices of same order, then $(AB)^{-1} = B^{-1} A^{-1}$

## Top Concepts

1. Order of a matrix gives the number of rows and columns present in the matrix.

2. If the matrix A has m rows and n columns then it is denoted by A $=[a_{ij}]_{m \times n}$ . $a_{ij}$ is i-j th or $(i, j)^{th}$ element of the matrix.

3. The simplest classification of matrices is based on the order of the matrix.

4. In case of a square matrix, the collection of elements $a_{11}$ , $a_{22,}$ and so on constitute the **Principal Diagonal** or simply the diagonal of the matrix Diagonal is defined only in case of square matrices.

$$\begin{bmatrix} a_{11} & a_{12} & --- & --- & a_{1n} \\ a_{21} & a_{22} & --- & --- & a_{2n} \\ --- & --- & --- & --- & --- \\ a_{i1} & a_{i2} & --- & --- & a_{in} \\ --- & --- & --- & --- & --- \\ a_{m1} & a_{m2} & --- & --- & a_{mn} \end{bmatrix}_{m \times n}$$

5. Two matrices of same order are comparable matrices.

6. If $A = \left[a_{ij}\right]_{m \times n}$ and $B = \left[b_{ij}\right]_{m \times n}$ are two matrices of order m × n, their sum is defined as a matrix $C = \left[c_{ij}\right]_{m \times n}$ where

$c_{ij} = a_{ij} + b_{ij}$ for $1 \le i \le m, 1 \le j \le n$

7.Two matrices can be added9or subtracted) if they are of same order. For multiplying two matrices A and B number of columns in A must be equal to the number of rows in B.

8. $A = \left[a_{ij}\right]_{m\,x\,n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k. Hence $kA = \left[ka_{ij}\right]_{m\,x\,n}$

9. If $A = \left[a_{ij}\right]_{m\,x\,n}$ and $B = \left[b_{ij}\right]_{m\,x\,n}$ are two matrices, their difference is represented as $A - B = A + (-1)B$.

10. Properties of matrix addition

(i) Matrix addition is commutative i.e A+B = B+A.

(ii) Matrix addition is associative i.e (A + B) + C = A + (B + C).

(iii) Existence of additive identity: Null matrix is the identity w.r.t addition of matrices

Given a matrix $A = \left[a_{ij}\right]_{m\,x\,n}$, there will be a corresponding null matrix O of same order such that A+O=O+A=A

(iv) The existence of additive inverse Let $A = [a_{ij}]_{m \times n}$ be any matrix, then there exists another matrix $-A = -[a_{ij}]_{m \times n}$ such that

$A + (-A) = (-A) + A = O.$

11. Properties of scalar multiplication of the matrices:

If $A = \left[a_{ij}\right], B = \left[b_{ij}\right]$ are two matrices, and k,L are real numbers then

(i) k(A +B) = k A + kB, (ii) (k + I)A = k A + I A

(ii) $k(A + B) = k([a_{ij}]+[b_{ij}]) = k[a_{ij}]+k[b_{ij}] = kA+kB$

(iii) $(k + L)A = (k + L)\left[a_{ij}\right] = \left[(k + L)a_{ij}\right] = k[a_{ij}] + L[a_{ij}] = kA + LA$

12. If $A = \left[a_{ij}\right]_{mxp}, B = \left[b_{ij}\right]_{pxn}$ are two matrices, their product AB, is given by $C = \left[c_{ij}\right]_{mxn}$ such that

$$c_{ij} = \sum_{k=1}^{p} a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{ip}b_{pj}.$$

In order to multiply two matrices A and B the number of columns in A = number of rows in B.

13. **Properties of Matrix Multiplication**
Commutative law does not hold in matrices, whereas the associative and distributive laws hold for matrix multiplication

(i) In general $AB \neq BA$

(ii) Matrix multiplication is associative $A(BC)=(AB)C$

(iii) Distributive laws:
$A(B+C)=AB+BC$;
$(A+B)C=AC+BC$

14. The multiplication of two non zero matrices can result in a null matrix.

15. Properties of transpose of matrices
(i) If A is a matrix, then $(A^T)^T=A$
(ii) $(A + B)^T = A^T + B^T$,
(iii) $(kB)^T = kB^T$, where k is any constant.

16. If A and B are two matrices such that AB exists then $(AB)^T = B^T A^T$

17. Every square matrix can be expressed as the sum of a symmetric and skew symmetric matrix i.e $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ for any square matrix A .

18. A square matrix A is called an orthogonal matrix when $AA^T=A^TA=I$.

19. A null matrix is both symmetric as well as skew symmetric.

20. Multiplication of diagonal matrices of same order will be commutative.

21. There are 6 elementary operations on matrices.Three on rows and 3 on columns. First operation is interchanging the two rows i.e $R_i \leftrightarrow R_j$ implies the $i^{th}$ row is interchanged with $j^{th}$ row. The two rows are interchanged with one another the rest of the matrix remains same.

22. Second operation on matrices is to multiply a row with a scalar or a real number i.e $R_i \to kR_i$ that $i^{th}$ row of a matrix A is multiplied by k.

23. Third operation is the addition to the elements of any row, the corresponding elements of any other row multiplied by any non zero number

i.e $R_i \to R_i + kR_j$ k multiples of jth row elements are added to ith row elements

24. Column operation on matrices are
(i) Interchanging the two columns: $C_r \leftrightarrow C_k$ indicates that rth column is interchanged with kth column

(ii) Multiply a column with a non zero constant i.e $C_i \to kC_i$
(iii) Addition of scalar multiple of any column to another column i.e $C_i \to C_i + kC_j$

25. Elementary operations helps in transforming a square matrix to identity matrix

26. Inverse of a square matrix, if it exists is unique.

27. Inverse of a matrix can be obtained by applying elementary row operations on the matrix A= IA. In order to use column operations write A=AI

28.Either of the two operations namely row or column operations can be applied. Both cannot be applied simultaneously

**Top Formulae**

1. An m x n matrix is a square matrix if m = n.
2. A = $[a_{ij}]$ = $[b_{ij}]$ = B if (i) A and B are of same order, (ii) $a_{ij} = b_{ij}$ for all possible values of i and j.
3. kA = $k[a_{ij}]_{m \times n} = [k(a_{ij})]_{m \times n}$.
4. - A = (-1) A
5. A – B = A + (-1) B

6. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ik}]_{n \times p}$, then $AB = C = [c_{ik}]_{m \times p}$, where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{ij}$

7. Elementary operations of a matrix are as follow:

i. $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$

ii. $R_i \to kR_i$ or $C_i \to kC_i$

iii. $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$