Class XII

Mathematics

Chapter:5

Continuity and Differentiability

Chapter Notes

Key Definitions

1. A function f(x) is said to be continuous at a point c if,
$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$
2. A real function f is said to be continuous if it is continuous at every point in the domain of f.
3. If f and g are real valued functions such that (f o g) is defined at c. If g is continuous at c and if f is continuous at g(c), then (f o g) is continuous at c.
4. A function f is differentiable at a point c if LHD=RHD
i.e $\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h} = \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$
5. Chain Rule of Differentiation: If f is a composite function of two functions u and v such that f = vou and t =u(x)
if both $\frac{dt}{dx}$ and $\frac{dv}{dx}$, exists then, $\frac{df}{dx} = \frac{dv}{dt}.\frac{dt}{dx}$
6. Logarithm of a to base b is xi.e $\log_b a$ =x if b^x = a where b > 1 be a real number. Logarithm of a to base b is denoted by $\log_b a$.

7. Functions of the form x = f(t) and y = g(t) are parametric functions.
8. **Rolle's Theorem**: If f : [a, b] $\to$ **R** is continuous on [a, b] and differentiable on (a, b) such that f (a) = f (b), then there exists some c in (a, b) such that f'(c) = 0

9. **Mean Value Theorem**: If f :[a, b] $\to$ **R** is continuous on [a, b] & differentiable on (a, b). Then there exists some c in (a, b) such that

$$f'(c) = \lim_{h\to 0}\frac{f(b)-f(a)}{b-a}$$

Key Concepts

1. A function is continuous at x = c if the function is defined at x = c and the value of the function at x = c equals the limit of the function at x = c.
2. If function f is not continuous at c, then f is discontinuous at c and c is called the point of discontinuity of f.
3. Every polynomial function is continuous.
4. Greatest integer function, [x] is not continuous at the integral values of x.
5. Every rational function is continuous.
6. Algebra of Continuous Functions

   Let f and g be two real functions continuous at a real number c, then

   (1) f + g is continuous at x = c

   (2) f − g is continuous at x = c

   (3) f. g is continuous at x = c

   (4) $\left(\frac{f}{g}\right)$ is continuous at x = c, (provided g(c) ≠ 0).
7. Derivative of a function f with respect to x is f′(x) which is given by

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

8. If a function f is differentiable at a point c, then it is also continuous at that point.
9. Every differentiable function is continuous but converse is not true.
10. Chain Rule is used to differentiate composites of functions.
11. **Algebra of Derivatives:**

If u & v are two functions which are differentiable, then

(i) $(u \pm v)' = u' \pm v'$ (Sum and Difference Formula)

(ii) $(uv)' = u'v + uv'$ (Product rule)

(iii) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ (Quotient rule)

12.**Implicit Functions**

If it is not possible to "separate" the variables x & y then function f is known as implicit function**.**

13.**Exponential function:** A function of the form y = f (x) = b^x where base b > 1

(1) Domain of the exponential function is R, the set of all real numbers.

(2) The point (0, 1) is always on the graph of the exponential function

(3) Exponential function is ever increasing

14.Properties of Logarithmic functions

(i)Domain of log function is R^+.

(ii) The log function is ever increasing

(iii) For x very near to zero, the value of log x can be made lesser than any given real number.

15.Logarithmic differentiation is a powerful technique to differentiate functions of the form f(x) = $[u(x)]^{v(x)}$. Here both f(x) and u(x) need to be positive.

16.**Logarithmic Differentiation**

$y=a^x$

Taking logarithm on both sides

$\log y = \log a^x$.

Using property of logarithms

$\log y = x \log a$

Now differentiating the implicit function

$$\frac{1}{y}.\frac{dy}{dx} = \log a$$

$$\frac{dy}{dx} = y \log a = a^x \log a$$

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17. A relation between variables x and y expressed in the form x=f(t) and y=g(t) is the parametric form with t as the parameter .Parametric equation of parabola $y^2=4ax$ is $x=at^2$,y=2at

18.**Parametric Differentiation:**

Differentiation of the functions of the form x = f(t) and y = g(t)

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$$

19.If y =f(x) and $\frac{dy}{dx}$ =f′(x) and if f′(x) is differentiable then

$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}$ or f′′ (x) is the second order derivative of y w.r.t x

Top Formulae

1. Derivative of a function at a point

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

2. Properties of Logarithms

$$\log(xy)=\log x+\log y$$

$$\log\left(\frac{x}{y}\right)=\log x-\log y$$

$$\log\left(x^y\right)=y\log x$$

$$\log_a x=\frac{\log_b x}{\log_b a}$$

3.Derivatives of Functions

- $\frac{d}{dx}x^n = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\text{cosec} x) = -\text{cosec} x \cot x$
- $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$
- $\frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}$
- $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}\left(\text{cosec}^{-1} x\right) = \frac{-1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}\left(e^x\right) = e^x$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$