Class XII: Mathematics
Chapter 6: Application of Derivatives
Chapter Notes

Key Concepts

1. Derivatives can be used to (i) determine rate of change of quantities(ii)to find the equation of tangent and normal(iii)to find turning points on the graph of a function(iv) calculate n^{th} root of a rational number (v) errors in calculations using differentials.

2. Whenever one quantity y varies with another x satisfying some rule y =f(x), then $\frac{dy}{dx}$ or f′(x) represents the rate of change of y with respect to x.
3. $\frac{dy}{dx}$ is positive if y and x increases together and it is negative if y decreases as x increases.
4. The equation of the tangent at (x_0, y_0) to the curve y = f (x) is:
   $y - y_0 = f'(x_0)(x - x_0)$
   Slope of a tangent = $\frac{dy}{dx} = \tan\theta$
5. The equation of the normal to the curve y = f (x) at (x_0, y_0) is:
   $(y-y_0)f'(x_0)+(x-x_0)= 0$
   Slope of Normal $= \frac{-1}{\text{slope of the tangent}}$

6. The angle of intersection of two curves is defined to be the angle between the tangents to the two curves at their point of intersection.

7. Let I be an open interval contained in domain of a real valued function f. Then f is said to be:

i. Increasing on I if $x_1 < x_2$ in I
   $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in I$
ii. Strictly increasing on I if $x_1 < x_2$ in I
   $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$
iii. Decreasing on I if $x_1 < x_2$ in I
   $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in I$
iv. Strictly decreasing on I if $x_1 < x_2$ in I
   $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$

8. Let f be a continuous function on [a,b] and differentiable on (a,b).Then
(a)f is increasing in[a,b] if f′(x)>0for each x∈(a,b)
(b) f is decreasing in[a,b] if f′(x)<0 for each x∈(a,b)
(c) f is constant in[a,b] if f′(x)=0 for each x∈(a,b)

9. Let f be a continuous function on [a,b] and differentiable on (a,b).Then
(a) f is strictly increasing in (a,b) if $f'(x)>0$for each $x \in (a,b)$
(b) f is strictly decreasing in (a,b) if $f'(x)<0$ for each $x \in (a,b)$
(c) f is constant in (a,b) if $f'(x)=0$ for each $x \in (a,b)$

10. A function which is either increasing or decreasing is called a monotonic function

11. Let f be a function defined on I.Then

a f is said to have a maximum value in I, if there exists a point c in I such that $f(c) > f(x)$, for all $x \in I$.

The number f (c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

b. f is said to have a minimum value in I, if there exists a point c in I such that $f(c) < f(x)$, for all $x \in I$.

The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.

c. f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I.

The number f (c) , in this case, is called an extreme value of f in I and the point c, is called an extreme point.

12. Every monotonic function assumes its maximum/ minimum value at the end points of the domain of definition of the function.

13. Every continuous function on a closed interval has a maximum and a minimum value

14. Derivative of a function at the point *c* represents the slope of tangent to the given curve at a point $x=c$.

15.If $f'(c)=0$ i.e. derivative at a point $x=c$ vanishes, which means slope of the tangent at $x=c$ is zero. Geometrically, this will imply that this tangent is parallel to x axis so $x=c$ will come out to be a turning point of the curve. Such points where graph takes a turn are called extreme points.

16.Let f be a real valued function and let c be an interior point in the domain of f. Then

a. c is called a point of local maxima if there is $h > 0$ such that

$f(c) > f(x)$, for all x in $(c - h, c + h)$

The value f (c) is called the local maximum value of f.

b. c is called point of local minima if there is an $h > 0$ such that

$f(c) < f(x)$, for all x in $(c - h, c + h)$

The value f (c) is called the local minimum value of f.

17. Let f be a function defined on an open interval I. Suppose $c \in I$ be any point. If f has a local maxima or a local minima at x = c, then either $f'(c) = 0$ or f is not differentiable at c.

18. **I Derivative Test:** Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

i. If $f'(x) > 0$ at every point sufficiently close to and to the left of c & $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima.
ii. If $f'(x) < 0$ at every point sufficiently close to and to the left of c, $f'(x) > 0$ at every point sufficiently close to and to the right of c, then c is a point of local minima.
iii. If $f'(x)$ does not change sign as x increases through c, then point c is called point of inflexion.

19. **II Derivative Test:** Let f be a function defined on an interval I & $c \in I$. Let f be twice differentiable at c. Then

i. x = c is a point of local maxima if $f'(c) = 0$ & $f''(c) < 0$.
ii. x = c is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$
iii. The test fails if $f'(c) = 0$ & $f''(c) = 0$.
By first derivative test, find whether c is a point of maxima, minima or a point of inflexion.

20. Working Rule to find the intervals in which the function f(x) increases or decreases

a) Differentiate f(x) first i.e. find f′(x)
b) Simplify f′(x) and factorise it if possible in case of polynomial functions.
c) Equate f′(x) to zero to obtain the zeroes of the polynomial in case of polynomial functions and angles in the given interval in case of trigonometric functions.
d) Divide the given interval or the real line into disjoint subintervals and then find the sign f′(x) in each interval to check whether f(x) is increasing or decreasing in a particular interval.

21. Let f be a continuous function on an interval I = [a, b]. Then f has the absolute maximum attains it at least once in I. Also, f has the absolute minimum value and attains of a function it at least once in I.

22. Let f be a differentiable function on a closed interval I and let c be any interior point of I. Then

a. $f'(c) = 0$ if f attains its absolute maximum value at c.

b. $f'(c) = 0$ if f attains its absolute minimum value at c.

23. Working Rule for finding the absolute maximum and minimum values in the interval [a,b]

Step 1: Find all critical points of f in the interval, i.e., find points x where either

$f'(x) = 0$ or f is not differentiable

Step 2: Take the end points of the interval.

Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify, the maximum and minimum values of f out of the values calculated in Step 3. This maximum and minimum value will be the absolute maximum (greatest) value f and the minimum value will be the absolute minimum (least) value of f.

24. Let $y = f(x)$, Δx be small increments in x and Δy be small increments in y corresponding to the increment in x, i.e., $\Delta y = f(x+\Delta x) - f(x)$.
Then

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x \text{ or } dy = \left(\frac{dy}{dx}\right)\Delta x \quad \Delta y \approx dy \text{ and } \Delta x \approx dx$$