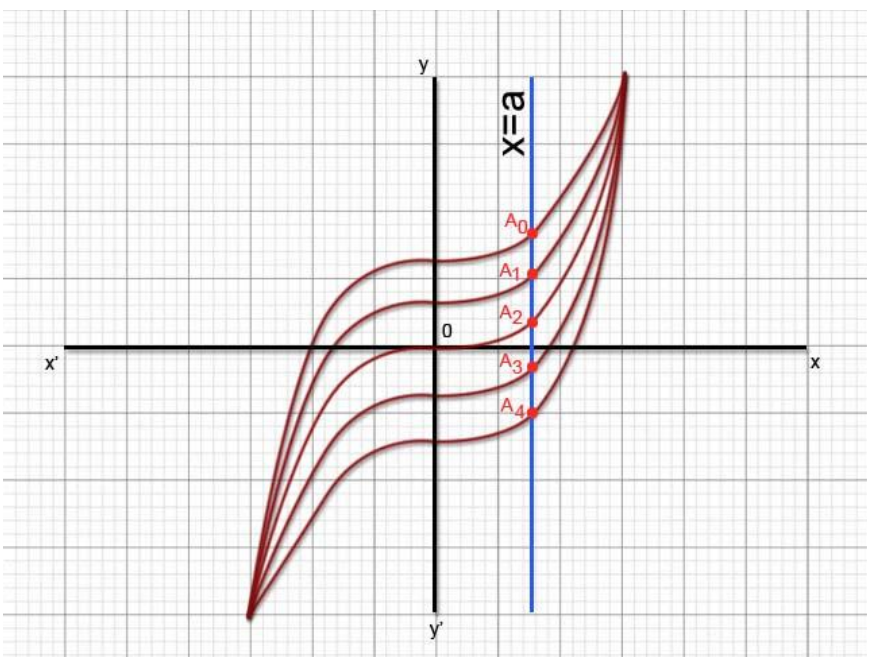
Class XII: Mathematics
Chapter 7: Integrals

Chapter Notes

Key Concepts

1. Integration is the inverse process of differentiation. The process of finding the function from its primitive is known as integration or anti differentiation.
2. Indefinite Integral $\int f(x)dx = F(x) + C$ where F(x) is the antiderivative of f(x).
3. $\int f(x)dx$ means integral of f w.r.t x ,f(x) is the integrand, x is the variable of integration, C is the constant of integration.
4. Geometrically indefinite integral is the collection of family of curves, each of which can be obtained by translating one of the curves parallel to itself.
   Family of Curves representing the integral of $3x^2$



$\int f(x)dx = F(x) + C$ represents a family of curves where different values of C correspond to different members of the family, and these members are obtained by shifting any one of the curves parallel to itself.

5. **Properties of antiderivatives:**

- $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$

- $\int kf(x)dx = k\int f(x)dx$ for any real number k
- $\int [k_1f_1(x) + k_2f_2(x) + ... + k_nf_n(x)]dx = k_1\int f_1(x)dx + k_2\int f_2(x)dx + ... + k_n\int f_n(x)dx$
  where, $k_1,k_2...k_n$ are real numbers & $f_1,f_2,..f_n$ are real functions

6.By knowing one anti-derivative of function f infinite number of anti derivatives can be obtained.

7.Integration can be done using many methods prominent among them are
(i)Integration by substitution
(ii)Integration using Partial Fractions
(iii)Integration by Parts
(iv) Integration using trigonometric identities

8. A change in the variable of integration often reduces an integral to one of the fundamental integrals. Some standard substitutions are
x^2+a^2 substitute x = a tan θ
$\sqrt{x^2-a^2}$ substitute x = a sec θ
$\sqrt{a^2-x^2}$ substitute x = a sin θ or a cos θ

9. A function of the form $\frac{P(x)}{Q(x)}$ is known as rational function. Rational functions can be integrated using Partial fractions.

10. **Partial fraction** decomposition or **partial fraction expansion** is used to reduce the degree of *either* the numerator or the denominator of a rational function.

11. **Integration using Partial Fractions**
A rational function $\frac{P(x)}{Q(x)}$ can be expressed as sum of partial fractions if $\frac{P(x)}{Q(x)}$ this takes any of the forms.

- $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$, a≠b
- $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$
- $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
- $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
- $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

where x^2+bx+c cannot be factorised further.

12.To find the integral of the function $\int \frac{dx}{ax^2+bx+c}$ or $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

ax^2+bx+c must be expressed as $a\left[\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right]$

13. To find the integral of the function $\int \frac{(px+q)dx}{ax^2+bx+c}$ or $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$; px+q

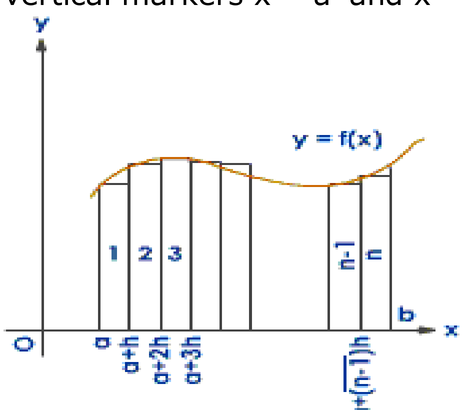
= A. $\frac{d}{dx}$(ax^2+bx+c)+B =A(2ax+b)+B

14.To find the integral of the product of two functions integration by parts is used.I and II functions are chosen using ILATE rule
I- inverse trigonometric
L- logarithmic A-algebra T-Trigonometric E-exponential , is used to identify the first function

14. Integration by parts:
The integral of the product of two functions = (first function) × (integral of the second function) – Integral of [(differential coefficient of the first function)× (integral of the second function)]

$\int f_1(x).f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left[\frac{d}{dx}f_1(x).\int f_2(x)dx\right]dx$ where f_1 & f_2 are functions of x.

15.Definite integral $\int_a^b f(x)dx$ of the function f(x) from limits a to b represents the area enclosed by the graph of the function f(x) the x axis, and the vertical markers x= 'a' and x = 'b'



16. **Definite integral as limit of sum:** The process of evaluating a definite integral by using the definition is called integration as limit of a sum or integration from first principles.

17. Method of evaluating $\int_a^b f(x)dx$

(i) Calculate anti derivative F(x)
(ii) calculate F(3) – F(1)

18. Area function

$A(x) = \int_a^x f(x)dx$, if x is a point in [a,b]

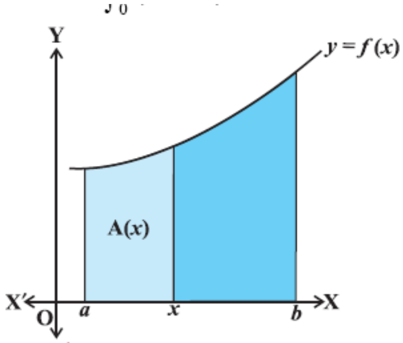


Fig 7.3

19. **Fundamental Theorem of Integral Calculus**

- **First Fundamental theorem** of integral calculus: If Area function,

  $A(x)= \int_a^x f(x)dx$ for all $x \geq a$, & f is continuous on [a,b].Then $A'(x)= f(x)$ for all $x \in [a, b]$.
- **Second Fundamental theorem** of integral calculus: Let *f* be a continuous function of x in the closed interval [*a*, *b*] and let F be antiderivative of $\frac{d}{dx}F(x) = f(x)$ for all x in domain of f, then

  $\int_a^b f(x)dx = \left[F(x) + C\right]_a^b = F(b) - F(a)$

Key Formulae

1.Some Standard Integrals

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

- $\int dx = x + C$
- $\int \cos x\, dx = \sin x + C$
- $\int \sin x\, dx = -\cos x + C$
- $\int \sec^2 x\, dx = \tan x + C$
- $\int \mathrm{co}\sec^2 x\, dx = -\cot x + C$
- $\int \sec x \tan x\, dx = \sec x + C$
- $\int \mathrm{co}\sec x \cot x\, dx = -\mathrm{cosec}\, x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \cot^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = -\mathrm{cos}\,\mathrm{ec}^{-1} x + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log a} + C$
- $\int \frac{1}{x} dx = \log|x| + C$
- $\int \tan x\, dx = \log|\sec x| + C$
- $\int \cot x\, dx = \log|\sin x| + C$
- $\int \sec x\, dx = \log|\sec x + \tan x| + C$
- $\int \mathrm{co}\sec x\, dx = \log|\mathrm{co}\sec x - \cot x| + C$

2.Integral of some special functions

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$
  - $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$
    - **Error!** = **Error!**
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3. Integration by parts

(i) $\int f_1(x).f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left[\frac{d}{dx}f_1(x).\int f_2(x)dx\right]dx$ where f_1 & f_2 are functions of x

(ii) $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+C$

4. Integral as a limit of sums:

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a) + f(a+h) + ... + f(a+(n-1)h)\right] \text{ where } h = \frac{b-a}{n}$$

5. Properties of Definite Integrals

- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$

  In particular, $\int_a^a f(x)dx = 0$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

- $$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{if } f(2a-x) = f(x)$$
  $$= 0 \qquad , \text{if } f(2a-x) = -f(x)$$

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx, \text{if } f(-x) = f(x)$$
$$= 0 \qquad , \text{if } f(-x) = -f(x)$$