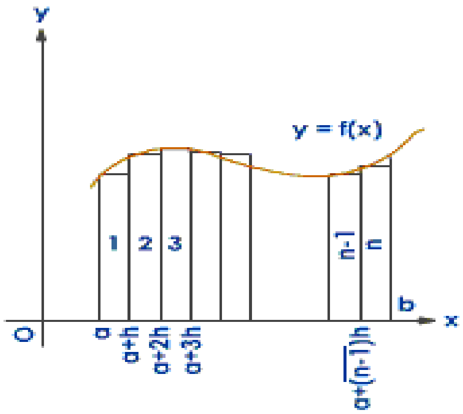
Class XII: Mathematics
Chapter 8: Applications of Integrals

Chapter Notes

Key Concepts

1. Definite integral $\int_a^b f(x)dx$ of the function f(x) from limits a to b represents the area enclosed by the graph of the function f(x) the x axis, and the vertical lines x= 'a' and x = 'b'



2. Area function is given by

$A(x) = \int_a^x f(x)dx$, where x is a point in [a, b]

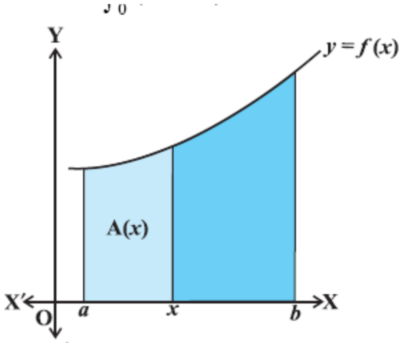
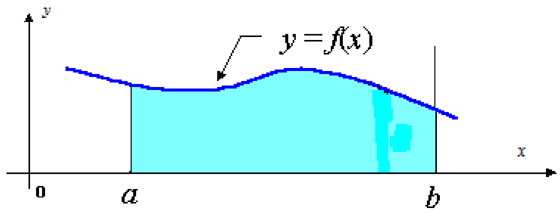


Fig 7.3

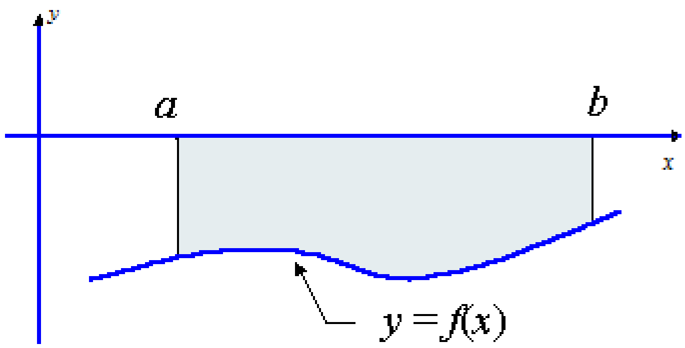
3. Area bounded by a curve, x-axis and two ordinates

Case 1: when curve lies above axis as shown below



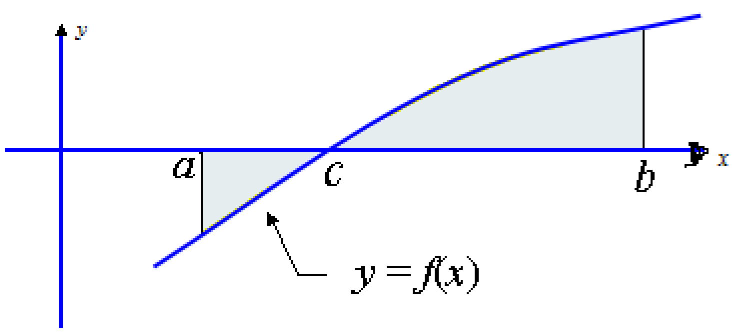
$$\text{Area} = \int_a^b f(x)dx$$

Case 2: Curves which are entirely below the x-axis as shown below



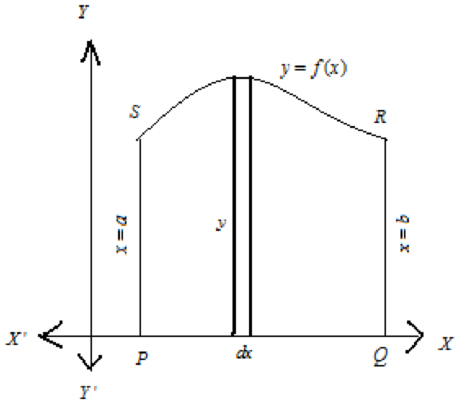
$$\text{Area} = \left|\int_a^b f(x)dx\right|$$

Case 3: Part of the curve is below the *x*-axis and part of the curve is above the *x*-axis.



$$\text{Area} = \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$$

4, area bounded by the curve y=f(x), the x-axis and the ordinates x=a and x=b using elementary strip method is computed as follows

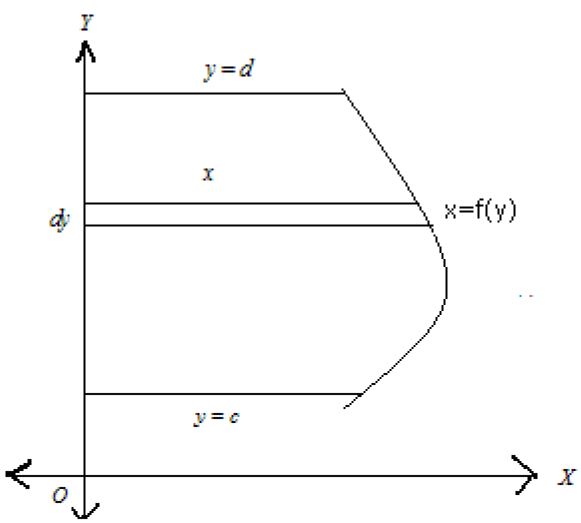


Area of elementary strip = y.dx

Total area = $\int_a^b dA = \int_a^b y dx = \int_a^b f(x)dx$

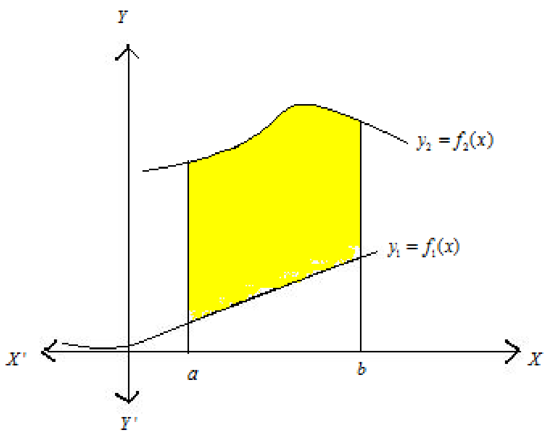
5. The area bounded by the curve x=f(y), the y-axis and the abscissa y=c and y=d is given by

$\int_c^d f(y)dy$ or, $\int_c^d x dy$



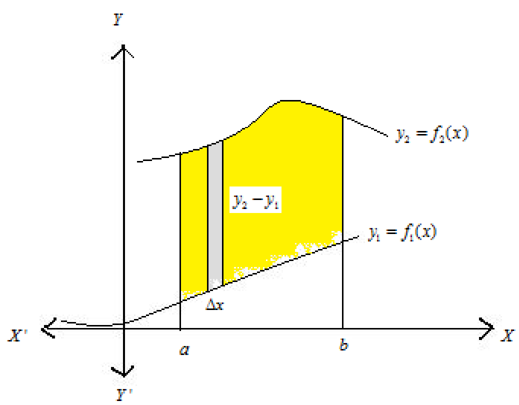
6. Area between $y_1 = f_1(x)$ and $y_2 = f_2(x)$, x = a and x = b is given by

$$\int_a^b y_2 dx - \int_a^b y_1 dx = \int_a^b (y_2 - y_1)dx$$



Area between two curves is the difference of the areas of the two graphs.

7. Area using strip



Each "typical" rectangle indicated has width Δx and height $y_2 - y_1$

Hence, Its area = $(y_2 - y_1)\,\Delta x$

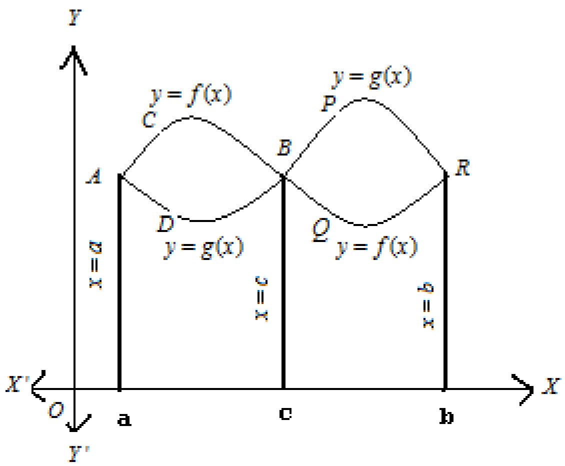
Total Area = $\sum_{x=a}^{b} (y_2 - y_1)\Delta x$

Area $= \int_a^b (y_2 - y_1)dx$

Area between two curves is also equal to integration of the area of an elementary rectangular strip within the region between the limits.

8. The area of the region bounded by the curve y = f (x), x-axis and the lines x = a and x = b (b > a) is Area$=\int_a^b ydx = \int_a^b f(x)dx$
9. The area of the region enclosed between two curves $y = f(x)$, $y = g(x)$ and
   the lines $x = a$, $x = b$ is
   Area $= \int_a^b [f(x) - g(x)]dx$ where, f(x) > g(x)
   in [a,b]

10. If f(x) ≥g(x) in [a, c] and f(x)≤g(x) in [c, b], where a< c< b



then the area of the regions bounded by curves is
Total Area= Area of the region ACBDA + Area of the region BPRQB

$$=\int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$$

Key Formulae

1. Some standard Integrals

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int dx = x + C$
- $\int \cos x\, dx = \sin x + C$

- $\int \sin x\,dx = -\cos x + C$
- $\int \sec^2 x\,dx = \tan x + C$
- $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
- $\int \sec x \tan x\,dx = \sec x + C$
- $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
- $\int \frac{dx}{1+x^2} = \cot^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
- $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log a} + C$
- $\int \frac{1}{x} dx = \log|x| + C$
- $\int \tan x\,dx = \log|\sec x| + C$
- $\int \cot x\,dx = \log|\sin x| + C$
- $\int \sec x\,dx = \log|\sec x + \tan x| + C$
- $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C$

2.Integral of some special functions

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$

- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$
  - $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$
    - **Error!** = **Error!**
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