Class XII: Mathematics
Chapter 9: Differential Equations
Chapter Notes

Key Concepts

1. An equation involving derivatives of dependent variable with respect to independent variable is called a **differential equation**.

   For example: $\dfrac{dy}{dx} = \cos x \quad \dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2x}$

2. Order of a differential equation is the order of the highest order derivative occurring in the differential equation.

   For example: order of $\dfrac{d^3y}{dx^3} + 3x(\dfrac{dy}{dx}) - 8y = 0 \quad$ is 3.

3. Degree of a differential equation is the   highest power (exponent) of the highest order derivative in it when it is written as a polynomial in differential coefficients.

   Degree of equation $\left(\dfrac{d^2y}{dx^2}\right)^3 + (c + b)\left(\dfrac{dy}{dx}\right)^4 = y \:$ is 3

4. Both order as well as the degree of differential equation are positive intgers.

5. A function which satisfies the given differential equation is called its solution.

6. The solution which contains as many arbitrary constants as the order of   the   differential   equation   is   called   a   **general   solution**.

7. The solution which is free from arbitrary constants is called **particular solution**.

8. Order of differential equation is equal to the number of arbitrary constants present in the general solution.

9. An nth order differential equation represents an n-parameter family of curves.

10. There are 3 Methods of Solving First Order, First Degree Differential Equations namely
    (i) Separating the variables if the variables can be separated.
    (ii) Substitution if the equation is homogeneous.
    (iii) Using integrating factor if the equation is linear different

11. **Variable separable** method is used to solve equations in which variables can be separated i.e terms containing y should remain with dy & terms containing x should remain with dx.

12. A differential equation which can be expressed in the form $\dfrac{dy}{dx} = f(x,y)$ or $\dfrac{dx}{dy} = g(x,y)$ where, f (x, y) & g(x, y) are homogenous functions is called a homogeneous differential equation.

13. Degree of each term is same in a homogeneous differential equation

14. A differential equation of the form $\dfrac{dy}{dx} + Py = Q$, where P and Q are constants or functions of x only is called a first order linear differential equation. If equation is of the form $\dfrac{dx}{dy} + Px = Q$ then P and Q are constants or functions of y

15. Steps to solve a homogeneous differential equation
$$\frac{dy}{dx} = F(x,y) = g\left(\frac{y}{x}\right) \qquad \ldots\ldots(1)$$

- Substitute y=v.x   …….(2)
- Differentiate (2) wrt to x
$$\frac{dy}{dx} = v + x\frac{dv}{dx} \qquad \ldots\ldots(3)$$
- Substitute & separate the variables
$$\frac{dv}{g(v) - v} = \frac{dx}{x}$$
- Integrate, $\displaystyle\int \frac{dv}{g(v) - v} = \int \frac{dx}{x} + C$

16. $\dfrac{dy}{dx} + Py = Q$ where, P and Q are constants
   or functions of x only
   Integrating factor (I.F)= $e^{\int Pdx}$
   Solution: y   (I.F)   =   ∫   (Q   ×   I.F)dx   +   C

17. $\dfrac{dx}{dy} + P_1y = Q_1$ where, P_1 & Q_1 are constants or functions of y only
   Integrating factor (I.F)= $e^{\int P1dy}$
   Solution: x (I.F) = ∫ (Q × I.F)dy + C