Ch: Alternating Current

Phy XII

Chapter Notes

TOP Formulae

Alternating Current (a.c)

The current whose magnitude changes with time and direction reverses periodically, is called alternating current.

a) Alternating emf E and current I at any time are given by:

$E = E_0 \sin\omega t$, where E_0 = NBAω

and $I = I_0 \sin(\omega t - \phi)$, here $I_0 = NBA\omega/R$

$\omega = 2\pi n = \frac{2\pi}{T}$, T $\rightarrow$ Time period

Values of Alternating Current and Voltage

a) Instantaneous value: It is the value of alternating current and voltage at an instant t.

b) Peak value: Maximum values of voltage E_0 and current I_0 in a cycle, are called peak values.

c) Mean value: For complete cycle.

$$< E > = \frac{1}{T}\int_0^T E\,dt = 0$$

$$< I > = \frac{1}{T}\int_0^T I\,dt = 0$$

Mean value for half cycle : $E_{mean} = \frac{2E_0}{\pi}$

d) Root – mean- square (rms) value:

$$E_{rms} = (< E^2 >)^{½} = \frac{E_0}{\sqrt{2}} = 0.707E_0 = 70.7\% E_0$$

$$\text{And } I_{rms} \; (< I^2 >)^{½} = \frac{I_0}{\sqrt{2}} = 0.707I_0 = 70.7\% I_0$$

RMS values are also called apparent or effective values.

Phase difference Between the EMF (Voltage) and the Current in an AC Circuit

a) For pure resistance: The voltage and the current are in same phase i.e. phase difference $\phi = 0$

b) For pure inductance: The voltage is ahead of current by $\pi/2$ i.e. phase difference $\phi = +\pi/2$.

c) For pure capacitance: The voltage lags behind the current by $\pi/2$ i.e. phase difference $\phi = -\pi/2$.

Reactance

a) Reactance $X = \frac{E}{I} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}} \pm \pi/2$

b) Inductive reactance

$X_L = \omega L = 2\pi nL$

c) Capacitive reactance

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi nC}$

Impedance

Impedance $Z = \frac{E}{I} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}} \phi$

Where ϕ is the phase difference of the voltage E relative to the current I.

b) For L – R series circuit:

$$Z_{RL} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega L^2}$$

And $\tan\phi = \left(\frac{\omega L}{R}\right)$ or $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

c) For R – C series circuit:

$$Z_{RC} = \sqrt{R^2 + X_c^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

And $\tan\phi = \frac{1}{\omega CR}$

Or $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$

d) For L – C series circuit:

$$Z_{LCR} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

And $\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$

Or $\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

Conductance

Reciprocal of resistance is called conductance.

$\therefore$ Conductance $\quad G = \frac{1}{R}$ mho

Power in and AC Circuit

a) Electric power = (current in circuit) x (voltage in circuit)

P = IE

b) Instantaneous power:

$P_{inst} = E_{inst} \times I_{inst}$

c) Average power:

$$P_{av} = \frac{1}{2}E_0 I_0 \cos\phi = E_{rms} I_{rms} \cos\phi$$

d) Virtual power (apparent power):

$$= \frac{1}{2}E_0 I_0 = E_{rms} I_{rms}$$

Power Factor

a) Power factor

$$\cos\phi = \frac{P_{av}}{P_v} = \frac{R}{Z}$$

b) For pure inductance

Power factor , $\cos\phi = 1$

c) For pure capacitance

Power factor , $\cos\phi = 0$

d) For LCR circuit

Power factor , $\cos\phi = \dfrac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

$$X = \left(\omega L - \frac{1}{\omega C}\right)$$

Wattless Current

The component of current differing in phase by $\pi/2$ relative to the voltage, is called wattles current.

rms value of wattless current :

$$= \frac{I_0}{\sqrt{2}} \sin\phi$$

$$= I_{rms} \sin\phi = \frac{I_0}{\sqrt{2}}\left(\frac{X}{Z}\right)$$

Choke Coil

An inductive coil used for controlling alternating current whose self inductance is high and resistance in negligible, is called choke coil.

The power factor of this coil is approximately zero.

Series Resonant Circuit

a) when the inductive reactance (X_L) becomes equal to the capacitive reactance (X_C) in the circuit, the total impedance becomes purely resistive (Z=R). In this state voltage and current are in same phase ($\phi = 0$), the current and power are maximum and impedance is minimum. This state is called resonance.

b) At resonance,

$$\omega_r L = \frac{1}{\omega_r C}$$

Hence resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$

c) In resonance the power factor of the circuit is one.

Half – Power Frequencies

Those frequencies f_1 and f_2 at which the power is half of the maximum power (power at resonance), i.e.,

$$P = \frac{1}{2} P_{max}$$

And $\quad I = \frac{I_{max}}{\sqrt{2}}$

f_1 and f_2 are called half – power frequencies

$$P = \frac{P_{max}}{2}$$

Band – Width

The frequency interval between half – power frequencies is called band – width.

$\therefore$ Bandwidth$\Delta f = f_2 - f_1$

For a series LCR resonant circuit,

$$\Delta f = \frac{1}{2\pi} \frac{R}{L}$$

Quality Factor (Q)

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

$$= \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

Or $\quad Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{f_r}{(f_2 - f_1)} = \frac{f_r}{\Delta f}$